DO NOT TURN OVER UNTIL INSTRUCTED TO DO SO.

NO CALCULATORS PERMITTED.

EXAM TIME IS 50 MINUTES.

THE EXAM CONSISTS OF 5 QUESTIONS.

Your name: ______________________________

Your SID: ______________________________

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1. Solve the following inequality. Express your answer as an interval.

$$\frac{x-1}{x+1} > 0$$

	$x < -1$	$-1 < x < 1$	$1 < x$
$x-1$	$-$	$-$	$+$
$x+1$	$-$	$+$	$+$
$\frac{x-1}{x+1}$	$+$	$-$	$+$

$\Rightarrow \quad \frac{x-1}{x+1} > 0$ for $x \in (-\infty, -1) \cup (1, \infty)$

2. Give an example of two functions $f, g : X \to X$, where X is a set you may choose, such that

$$f \circ g \neq g \circ f \tag{1}$$

Take $f: \mathbb{R} \to \mathbb{R}$, $f(x) = 2x$

$g: \mathbb{R} \to \mathbb{R}$, $g(x) = x+1$

$\Rightarrow$ $(f \circ g)(x) = f(g(x)) = 2(x+1) = 2x+2$

$(g \circ f)(x) = g(f(x)) = 2x+1$

$\Rightarrow$ $g \circ f \neq f \circ g$.

3. (a) If a line L has slope $m \neq 0$, what is the slope of a line L'
   i. perpendicular to L
   ii. parallel to L

   (b) Sketch the line $L : y = 2x + 1$ and point $P : (2, 0)$

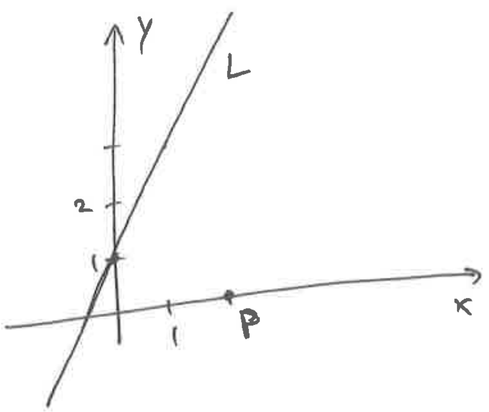
   (c) Write down the equation for the line L' perpendicular to L passing through P

   (d) Find the coordinates of the point of intersection of L with L'

(a) (i) has slope $-1/m$

(ii) has slope m

(b)



(c) L': slope $-\frac{1}{2}$

$\Rightarrow \quad y - 0 = -\frac{1}{2}(x-2)$

$\Rightarrow \quad y = -\frac{1}{2}x + 1$

(d) Need to find (x, y) st $\begin{array}{l} y = 2x+1 \\ y = -\frac{1}{2}x+1 \end{array}$

$\Rightarrow \quad 1 + 2x = -\frac{1}{2}x + 1$

$\Rightarrow \quad x = 0$

$\Rightarrow \quad y = 1$

$\Rightarrow$ intersection point at $(0, 1)$.

4. Let $f(x) = |x+1| - |x-1|$

   (a) Plot the graph of $f(x)$

   (b) Is $f(x)$ a one-to-one function? Justify your answer.

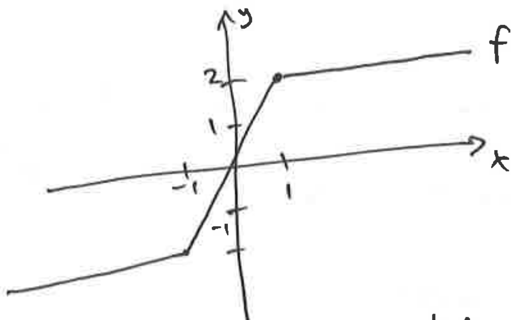
   (c) Is $f(x)$ an even or odd function? Justify your answer.

   (d) Plot the graph of $\frac{1}{2}f(x+2)+2$ using simple transformations of $f(x)$

(a)
$$f(x) = \begin{cases} -(x+1) - (-(x-1)) & \text{for } x < -1 \\ (x+1) - (-(x-1)) & \text{for } -1 \le x < 1 \\ (x+1) - (x-1) & \text{for } 1 \le x \end{cases}$$

$$= \begin{cases} -2 & \text{for } x < -1 \\ 2x & \text{for } -1 \le x < 1 \\ 2 & \text{for } 1 \le x \end{cases}$$

Plot:



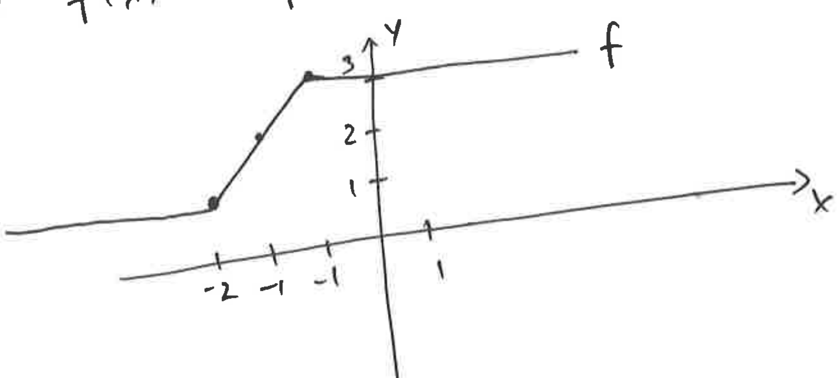
(b): No! Doesn't pass horizontal line test.

(c) odd function! : $f(-x) = |-x+1| - |-x-1|$

$= |x-1| - |x+1|$

$= -f(x)$.

(d) $f(x) \xrightarrow[\text{by } 2]{\text{left shift}} f(x+2) \xrightarrow[\text{by } \frac{1}{2}]{\text{vertical stretch}} \frac{1}{2}f(x+2) \xrightarrow[\text{by } 2]{\text{shift up}} \frac{1}{2}f(x+2)+2$



5. (a) Complete the square of $x^2 + 2(a-1)x + a^2$ and find the coordinates of the vertex of the parabola, when $x^2 + 2(a-1)x + a^2$ is viewed as a function in x

(b) By (a) or another method find all real numbers a such that the quadratic equation $x^2 + 2(a-1)x + a^2 = 0$ in x has

i. No solution in x

ii. One solution in x

iii. Two solutions in x

(a) Complete the square $x^2 + 2(a-1)x + (a-1)^2 - (a-1)^2 + a^2$

$$= (x + a - 1)^2 - a^2 + 2a - 1 + a^2$$

$$= (x + a - 1)^2 + (2a - 1)$$

$\Rightarrow$ vertex $(1-a, 2a-1)$

(b) Vertex above x-axis $\Rightarrow$ no solution: So $2a-1>0$, $a > 1/2$ no solution

vertex on x-axis $\Rightarrow$ one solution: So $2a-1=0$, $a = 1/2$ one solution

vertex below x-axis $\Rightarrow$ two solutions So $2a-1<0$, $a<1/2$, two solutions

Alternatively look at determinant: $4(a-1)^2 - 4a^2 = 4(a^2 - 2a + 1 - a^2)$

$$= 4(1-2a)$$

END OF EXAM