

Solutions Worksheet 2

-1-

1)

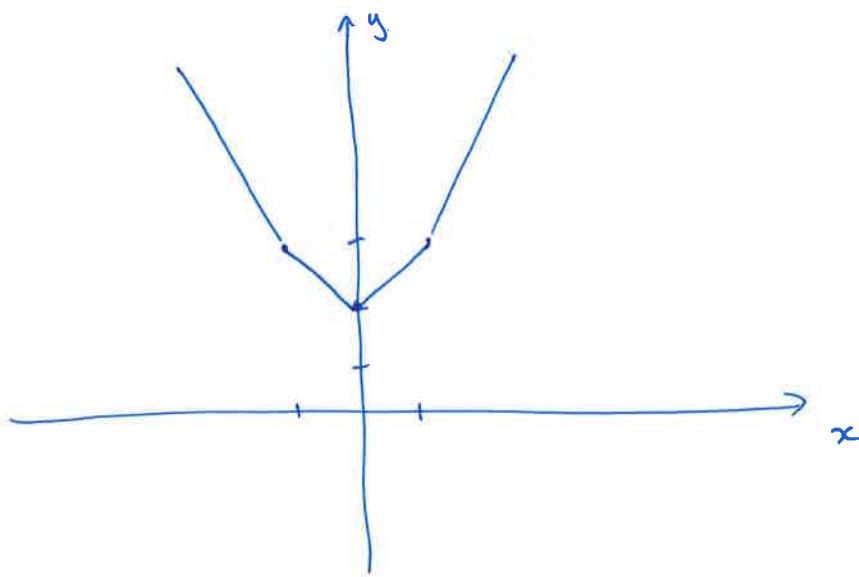
a) For  $x < -1$  :  $f(x) = -(x-1) - (x) - (x+1)$   
 $= -3x$

For  $-1 \leq x \leq 0$   $f(x) = -(x-1) - (x) + (x+1)$   
 $= -x + 2$

For  $0 \leq x \leq 1$  :  $f(x) = -(x-1) + (x) + (x+1)$   
 $= 1x + 2$

For  $1 \leq x$  :  $f(x) = (x-1) + (x) + (x+1)$   
 $= 3x$

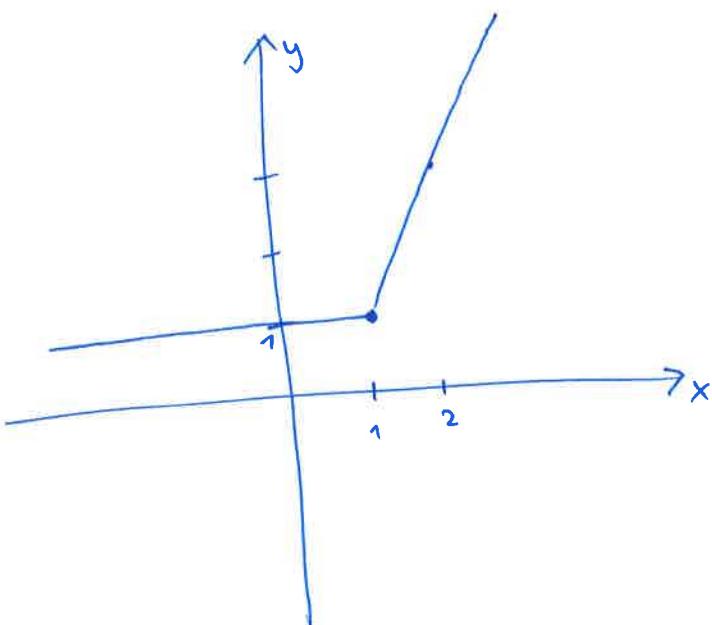
Plot:



b) For  $x \geq 1$  :  $f(x) = |x-1+x| = |2x-1| = 2x-1$   
 $\wedge$  positive for  $x \geq 1$

For  $x < 1$  :  $f(x) = |- (x-1) + x| = |1| = 1$

Plot

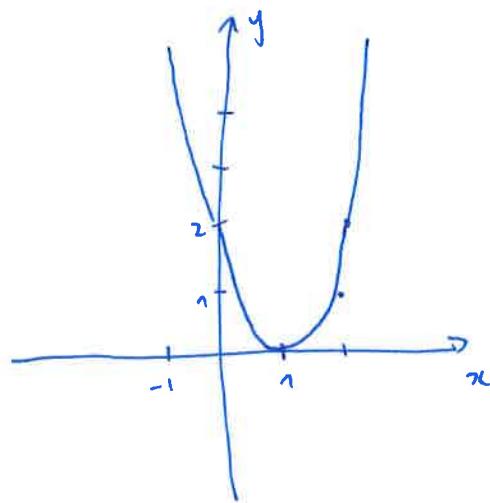


c)  $f(x) = 2x^2 - 4x + 2$

$$= 2(x^2 - 2x + 1)$$

$$= 2(x-1)^2$$

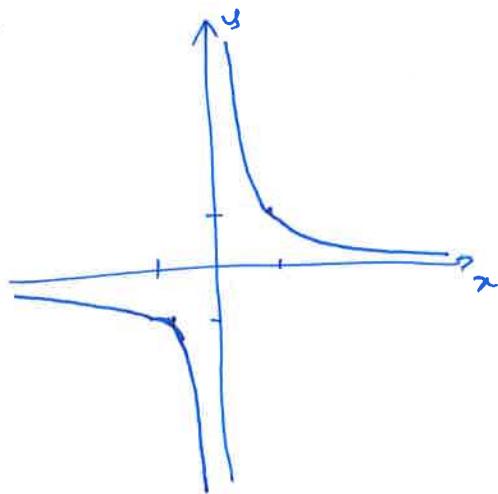
Plot:



2) a) domain :  $\mathbb{R} \setminus \{0\}$

range :  $\mathbb{R} \setminus \{0\}$

b) Plot:

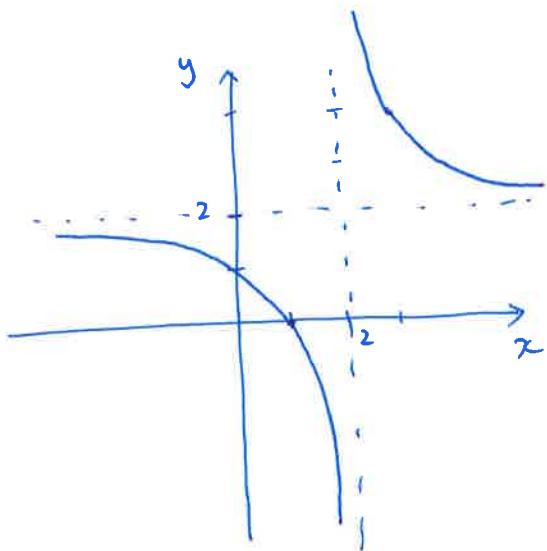


c)

$$\frac{x-a}{x-b} = \frac{x-b+b-a}{x-b} = \frac{x-b}{x-b} + \frac{b-a}{x-b} = 1 + \frac{b-a}{x-b}$$

d)  $g(x) = 2 \left( \frac{x-1}{x-2} \right) = 2 \left( 1 + \frac{1}{x-2} \right) = 2 \frac{1}{x-2} + 2$

Plot:

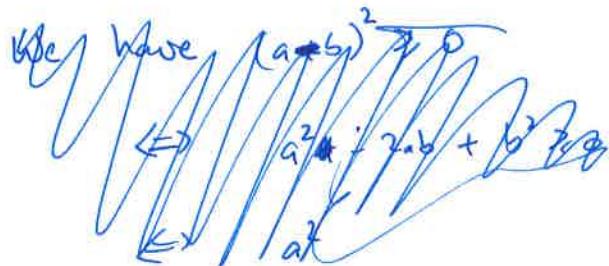


$$3a) (a+b)^2 = (a+b)(a+b) = a^2 + ab + ba + b^2 = a^2 + 2ab + b^2$$

$$b) (a-b)^2 = (a-b)(a-b) = a^2 - ab - ba + b^2 = a^2 - 2ab + b^2$$

$$c) (a-b)(a+b) = a^2 + ab - ba - b^2 = a^2 - b^2$$

d)



We have  $(a-b)^2 \geq 0$  for all  $a, b \geq 0$

$$\Leftrightarrow a^2 - 2ab + b^2 \geq 0$$

$$\Leftrightarrow a^2 - 2ab + b^2 + 4ab \geq 4ab$$

$$\Leftrightarrow a^2 + 2ab + b^2 \geq 4ab$$

$$\Leftrightarrow (a+b)^2 \geq 4ab$$

$$\Leftrightarrow a+b \geq 2\sqrt{ab} \quad (\text{taking squares of both sides})$$

$$\Leftrightarrow \frac{a+b}{2} \geq \sqrt{ab}$$

e) We have  $(a-b)^2 \geq 0$  for all  $a, b \geq 0$

$$\Leftrightarrow a^2 - 2ab + b^2 \geq 0$$

$$\Leftrightarrow a^2 + b^2 \geq 2ab$$

$$\Leftrightarrow \frac{a^2 + b^2}{2} \geq ab$$

$$\Leftrightarrow \sqrt{\frac{a^2 + b^2}{2}} \geq \sqrt{ab}$$

f) We have  $(a-b)^2 \geq 0$  for all  $a, b \geq 0$

$$\Leftrightarrow a^2 - 2ab + b^2 \geq 0$$

$$\Leftrightarrow a^2 + b^2 \geq 2ab$$

$$\Leftrightarrow 2a^2 + 2b^2 \geq a^2 + b^2 + 2ab \quad (\text{Add } a^2 + b^2 \text{ on both sides})$$

$$\Leftrightarrow \frac{a^2 + b^2}{2} \geq \frac{a^2 + 2ab + b^2}{4} \quad (\text{divide by 4 on both sides})$$

$$\Leftrightarrow \sqrt{\frac{a^2 + b^2}{2}} \geq \sqrt{\frac{a^2 + 2ab + b^2}{4}}$$

$$\Leftrightarrow \sqrt{\frac{a^2 + b^2}{2}} \geq \sqrt{\frac{(a+b)^2}{4}}$$

$$\Leftrightarrow \sqrt{\frac{a^2 + b^2}{2}} \geq \frac{a+b}{4} \quad (as a, b \geq 0, \text{ don't need absolute value signs})$$

g) We have  $a^2 + b^2 \leq \max(a, b)^2 + \max(a, b)^2 = 2 \max(a, b)^2$

hence  $\frac{a^2 + b^2}{2} \leq \max(a, b)^2$

hence  $\sqrt{\frac{a^2 + b^2}{2}} \leq \max(a, b)$

Similarly we have  $ab \geq \min(a, b) \min(a, b) = \min(a, b)^2$

Thus  $\sqrt{ab} \geq \min(a, b)$ .

We therefore get the desired chain of inequalities.