

Solutions Worksheet 1

-1-

1) a) $x^2 + 2x - 1 > 3$

$$\Leftrightarrow x^2 + 2x - 3 > 0$$

$$x^2 + 2x - 3 = 0 \Leftrightarrow x_{\pm} = \frac{-2 \pm \sqrt{4+12}}{2} = 1/-3$$

$$\text{Thus } x^2 + 2x - 3 > 0 \Leftrightarrow -3 < x < 1$$

b) $\frac{(x-1)(x-2)}{(x-3)(x-4)} > 0 \Leftrightarrow (x-1)(x-2) > 0 \text{ and } (x-3)(x-4) > 0$
 or $(x-1)(x-2) < 0 \text{ and } (x-3)(x-4) < 0$

Now $(x-1)(x-2) > 0 \Leftrightarrow x < 1 \text{ or } x > 2 \quad (\mathbb{R} \setminus [1,2])$

$$(x-3)(x-4) > 0 \Leftrightarrow x < 3 \text{ or } x > 4 \quad (\mathbb{R} \setminus [3,4])$$

$$(x-1)(x-2) < 0 \Leftrightarrow 1 < x < 2 \quad ((1,2))$$

$$(x-3)(x-4) < 0 \Leftrightarrow 3 < x < 4 \quad ((3,4))$$

So our solution is $\left[(\mathbb{R} \setminus [1,2]) \cap (\mathbb{R} \setminus [3,4]) \right] \cup ((1,2) \cap (3,4))$

$$= \mathbb{R} \setminus ([1,2] \cup [3,4]) \cup \emptyset$$

$$= \mathbb{R} \setminus ([1,2] \cup [3,4])$$

$$= (-\infty, 1) \cup (2, 3) \cup (4, \infty)$$

c) $|x+1| + |x-1| > 3$

Case 1 ($x < -1$)

$$|x+1| + |x-1| = -(x+1) - (x-1)$$
$$= -2x$$

so we need $-2x > 3$ or $x < -\frac{3}{2}$

Case 2 ($-1 \leq x \leq 1$)

$$|x+1| + |x-1| = (x+1) - (x-1) = 2 \neq 3$$

so no x in this range solve the inequality.

Case 3 ($x > 1$)

$$|x+1| + |x-1| = (x+1) + (x-1) = 2x$$

$$\text{so we } 2x > 3 \Leftrightarrow x > \frac{3}{2}$$

Putting all of above together we get solution

$$(-\infty, -\frac{3}{2}) \cup (\frac{3}{2}, \infty)$$

2) a) $x^2 - 5x + 6 = 0 \Leftrightarrow x=2 \text{ or } x=3$

(use quadratic formula or vieta)

b) $\frac{x-1}{x+2} = 3 \Leftrightarrow x-1 = 3(x+2)$
 $\Leftrightarrow x-1 = 3x+6$
 $\Leftrightarrow -7 = 2x$
 $\Leftrightarrow x = -\frac{7}{2}$

c) case 1: ($x > 0$)

$$\Rightarrow x + \frac{x}{2} = -4$$

$$\Rightarrow \frac{3x}{2} = -4$$

$$\Rightarrow x = -\frac{8}{3}$$

however $-\frac{8}{3} \neq 0$, so no solution in this case!

case 2: ($x \leq 0$)

$$\Rightarrow x - \frac{x}{2} = -4$$

$$\Rightarrow \frac{x}{2} = -4$$

$$\Rightarrow x = -8$$

so only solution is $x = -8$

$$d) | |x+1| - 1 | = \frac{1}{2}$$

Case 1 $(x+1 > 0)$

$$\Rightarrow | |x+1| - 1 | = | (x+1) - 1 | = |x| = \frac{1}{2}$$

$$\Rightarrow \text{so } x = \frac{1}{2} \text{ or } x = -\frac{1}{2} \quad (\text{Both solutions as we only require } (x+1) > 0 \text{ in this case})$$

case 2 $(x+1 \leq 0)$

$$\Rightarrow | |x+1| - 1 | = | -(x+1) - 1 | = | -x - 2 | \\ = |x+2| = \frac{1}{2}$$

$$\text{So we need } x = -\frac{3}{2} \text{ or } x = -\frac{5}{2}$$

since both $x = -\frac{3}{2}$ and $x = -\frac{5}{2}$ satisfy $x+1 \leq 0$,
both are solutions

Putting all solutions together we get $x = -\frac{5}{2}, -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}$.

3)

a) take $f: \mathbb{R} \rightarrow [0, \infty)$, defined by $f(x) = x^2$

b) take $f: \{A, B, C, \dots, X, Y, Z\} \rightarrow \mathbb{N}$

where $f(A) = 1, f(B) = 2, f(C) = 3, \dots, f(X) = 24, f(Y) = 25$
 $f(Z) = 26$.

c) take $f: \mathbb{R} \rightarrow \mathbb{R}$, defined by $f(x) = x^2$

4) Assume $\sqrt{3}$ were rational.

Then $\sqrt{3} = \frac{a}{b}$ for some $a, b \in \mathbb{N}, b \neq 0$.

Squaring we get $3 = \frac{a^2}{b^2} \Leftrightarrow 3b^2 = a^2$

But then a is divisible by 3, so $a = 3a'$ for
 a' some other integer.

So our equation becomes $3b^2 = (3a')^2 \Leftrightarrow b^2 = 3a'^2$

But then b must also be divisible by 3. So $b = 3b'$

Therefore we get $(3b')^2 = 3a'^2 \Leftrightarrow 3b'^2 = a'^2$.

So we get $\sqrt{3} = \frac{a'}{b'}$, for $a' < a$ and $b' < b$.

But now we can repeat this process indefinitely,
which is a contradiction (can't repeat process infinitely
often because either a or b will become zero, which doesn't
make sense). Hence we can't write $\sqrt{3} = \frac{a}{b}$. q.e.d.