Your name: _____

Your SID: _____

Question 1	/ 20
Question 2	/ 20
Question 3	/ 20
Question 4	/ 20
Question 5	/ 20
Total	/ 100

1. Solve the following inequality. Express your answer as an interval.

$$\frac{x-1}{x+1} > 0$$

$x - 1:$ $-$ $-$ $+$

$x + 1:$ $-$ $+$ $+$

-1 0 1
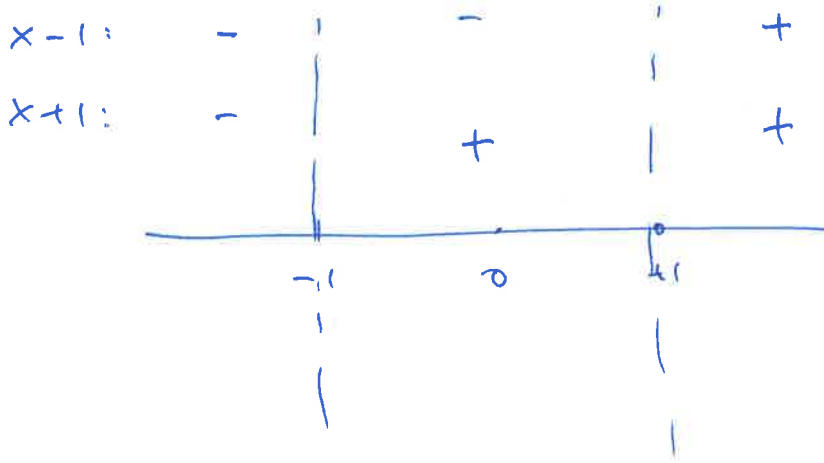
Hence $\dfrac{x-1}{x+1} > 0$ on $(-\infty, -1) \cup (1, \infty)$

2. Consider the polynomials

$$p(x) = 3x^2 - 1$$
$$q(x) = x^3 + 2x + 1$$

Write the composition $(p \circ q)(x)$ in expanded form, i.e. in the form $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$, for some n and numbers $a_0, a_1, ..., a_n$. Is $(p \circ q)(x)$ a polynomial?

$$(p \circ q)(x) = p(q(x)) = 3(x^3 + 2x + 1)^2 - 1$$

$$= 3(x^3 + 2x + 1)(x^3 + 2x + 1) - 1$$

$$= 3(x^6 + 2x^4 + x^3 + 2x^4 + 4x^2 + 2x$$
$$+ x^3 + 2x + 1) - 1$$

$$= 3x^6 + 12x^4 + 6x^3 + 12x^2 + 12x + 2$$

Yes, $(p \circ q)(x)$ is a polynomial!

3. (a) If a Line L has slope $m \neq 0$, what is the slope of a line L'
     i. perpendicular to L
     ii. parallel to L

   (b) Sketch the Line $L : y = 2x + 1$ and point $P : (2, 0)$

   (c) Write down the equation for the line L' perpendicular to L passing through P

   (d) Find the coordinates of the point of intersection of L with L'

(a) (i) slope $-\frac{1}{m}$

   (ii) slope m

(b)
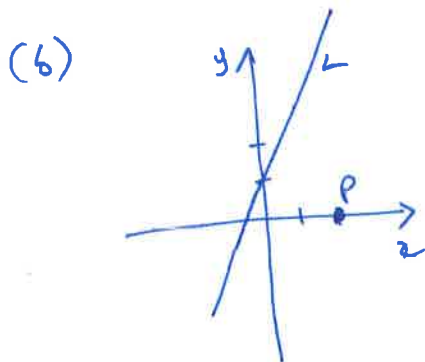
(c) L' has slope $-\frac{1}{2}$. Hence $L': \quad y - 0 = -\frac{1}{2}(x - 2)$

$$\Rightarrow \quad y = -\frac{1}{2}x + 1$$

(d) We have

$$-\frac{1}{2}x + 1 = 2x + 1$$

$$\Rightarrow \quad 0 = \frac{5}{2}x$$

$$\Rightarrow \quad x = 0$$

$$\Rightarrow \quad y = 2 \cdot 0 + 1 = 1$$

$\Rightarrow$ intersection at point $(0, 1)$.

TURN OVER

4. Let $f(x) = |x+1| - |x-1|$
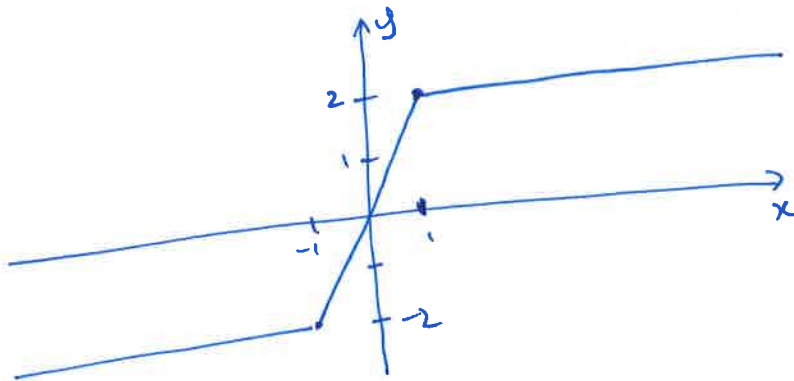
    (a) Plot the graph of $f(x)$

    (b) Is $f(x)$ a one-to-one function? Justify your answer.

    (c) Is $f(x)$ an even or odd function? Justify your answer.

    (d) Plot the graph of $\frac{1}{2}f(x+2) + 2$ using simple transformations of $f(x)$

(a) on $\quad x < -1$: $\quad f(x) = -(x+1) - (-(x-1)) = -2$

   on $-1 \le x \le 1$: $\quad f(x) = (x+1) + (x-1) = 2x$

   on $\quad x > 1$: $\quad f(x) = (x+1) - (x-1) = 2$
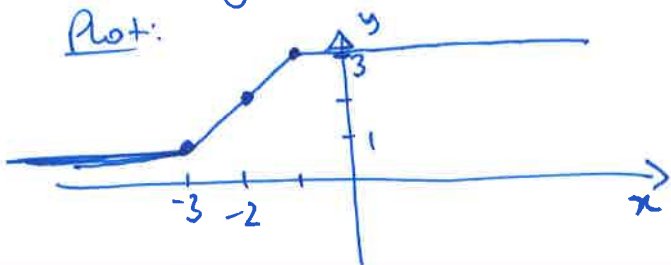
<u>Plot</u>

(b) No, by the horizontal line test

(c) $f(x)$ is odd since $\quad f(-x) = |-x+1| - |-x-1|$

$$= |x-1| - |x+1|$$

$$= -(|x+1| - |x-1|)$$

$$= -f(x)$$

$f(x)$ is not even! Graph is not symmetric!

(d) Graph of $\frac{1}{2}f(x+2) + 2$ is the graph of $f(x)$ shifted left by 2, scaled vertically by factor of $\frac{1}{2}$ and shifted upwards by 2

<u>Plot:</u>

TURN OVER

5. (a) For which fixed a does $x^2 + 2(a-1)x + a^2 = 0$ have

     i. No solution in x

     ii. One solution in x

     iii. Two solutions in x

  (b) Complete the square of $x^2 + 2(a-1)x + a^2$ and find the coordinates of the vertex of the parabola, when $x^2 + 2(a-1)x + a^2$ is viewed as a function in x

(a) We have $D = {}^{"}b^2 - 4ac{}^{"} = \left(2(a-1)\right)^2 - 4a^2$

$$= 4(a^2 - 2a + 1) - 4a^2$$

$$= -8a + 4$$

So (i) if $D < 0 \Leftrightarrow -8a + 4 < 0 \Leftrightarrow a > \tfrac{1}{2}$

(ii) if $D = 0 \Leftrightarrow a = \tfrac{1}{2}$

(iii) if $D > 0 \Leftrightarrow a < \tfrac{1}{2}$

(b) $\quad x^2 + 2(a-1)x + a^2 = x^2 + 2(a-1)x + (a-1)^2 - (a-1)^2 + a^2$

$$= (x + a - 1)^2 - a^2 + 2a - 1 + a^2$$

$$= (x + a - 1)^2 + 2a - 1$$

$\Rightarrow$ Vertex at $(1 - a, \; 2a - 1)$

END OF EXAM

BLANK PAGE

BLANK PAGE

BLANK PAGE

BLANK PAGE