

## Addendum to composition of functions

We were given functions  $f(x) = x+2$  and  $g(x) = \sqrt{x}$ .

Since no domain or codomain is given, we assume

- $f$  has domain  $\mathbb{R}$  and codomain  $\mathbb{R}$
- $g$  has domain  $\mathbb{R}_{\geq 0}$  (positive reals) and codomain  $\mathbb{R}$ .

So we have:  $f: \mathbb{R} \rightarrow \mathbb{R}$  and  $g: \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}$

Hence at first we can only compose  $f \circ g: \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}$  and not  $g \circ f$  since the codomain of  $f$ , which is  $\mathbb{R}$ , is not included in the domain of  $g$ , which is  $\mathbb{R}_{\geq 0}$ .

To define  $g \circ f$  we need to restrict the domain of  $f$  s.t. the range of  $f$  is included in the domain of  $g$ . If we take  $f: [-2, \infty) \rightarrow \mathbb{R}_{\geq 0}$

then the range of  $f$  is  $\mathbb{R}_{\geq 0}$  (since  $x+2 \geq 0$  for  $x \in [-2, \infty)$ ).

So we get  $g \circ f: [-2, \infty) \rightarrow \mathbb{R}$  the composite function

with  $(g \circ f)(x) = g(f(x)) = g(x+2) = \sqrt{x+2}$  (Note  $\sqrt{x+2}$  only makes sense on  $[-2, \infty)$ !)