Worksheet 8

1) a) Use the stretching principle to show that for $a, b, k > 0$ and $a \le b$ we have area $(\frac{1}{x}, a, b) =$ area $(\frac{1}{x}, ak, bk)$

b) For $a > b > 0$ we define area $(\frac{1}{x}, a, b) = -$ area $(\frac{1}{x}, b, a)$
Use this definition and part (a) to show area $(\frac{1}{x}, 1, c^{-1}) = -$ area $(\frac{1}{x}, 1, c)$

c) Use Example 6, p. 298 to carefully explain why
area $(\frac{1}{x}, 1, c^t) = t$ area $(\frac{1}{x}, 1, c)$ for every $c > 1$ and $t > 0$

d) Use part b) and c) to show area $(\frac{1}{x}, 1, c^t) = t$ area $(\frac{1}{x}, 1, c)$ for all $c > 0$ and $t \in \mathbb{R}$

e) Explain why d) proves that area $(\frac{1}{x}, 1, c) = \ln(c)$

2) Instead of approximating $\ln(1+t) =$ area $(\frac{1}{x}, 1, 1+t)$ by one rectangle as in lectures, approximate it by one trapezoid. Compare the approximation formula you obtain with the approximation $\ln(1+t) \approx t$ for small t

3) a) Let $n \geq 1$ be an integer and $x > 0$. Use compound interest to explain why

$$1 + x \le \left(1 + \frac{x}{n}\right)^n \le e^x$$

b) Plot $1+x$, $\left(1+\frac{x}{2}\right)^2$ and e^x in one coordinate system

4a) Approximate area $(\frac{1}{x}, 1, n+1)$ by n rectangles of width 1 above to get an approximation of $\ln(n+1)$

b) What does a) say about $1 + \frac{1}{2} + \cdots + \frac{1}{n}$ for n large?