Worksheet 4

1) Show that for $p(x)$ and $q(x)$ non-zero polynomials the following holds
   a) $p(x)\,q(x)$ is a polynomial
   b) $\deg(pq) = \deg p + \deg q$
   c) $\deg(p+q) \leq \text{maximum}\{\deg p, \deg q\}$

2) What is the ~~area~~ maximum area a rectangle of given circumference L can have (express answer in terms of L)

3) Show that $x^3 - 3x + 1$ has three real roots

4) Given that $x^3 + 6x^2 + 11x - 6$ has root $x = 1$, find the other roots

5) Use polynomial division to compute the quotient $\frac{x^5 - 1}{x - 1}$

6) Find the roots of $x^4 - 4x^2 + 3$ (Hint: Use substitution $u = x^2$)

7) Recall Pythagoras' Theorem for right triangles ($a^2 + b^2 = c^2$)
   a) Use phythagoras to show that the distance between points $P_1 : (x_1, y_1)$ and $P_2 : (x_2, y_2)$ is given by $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
   b) Let the point $P : (2, 3)$ and line $L : y = 2 - x$ be given. Find the point P' on the line L which has the least distance from P
   c) Show that the line L' given by P and P' is perpendicular to L.

8) Show that every polynomial of odd degree has at least one real root.