WORKSHEET: HOMEWORK 2

1) Sketch the graphs of the following functions

   a) $f(x) = |x-1| + |x| + |x+1|$

   b) $f(x) = | |x-1| + x |$

   c) $f(x) = 2x^2 - 4x + 2$

2) Let $f(x) = \dfrac{1}{x}$.

   a) What are the domain and range of f?

   b) Plot the graph of f

   c) Show that $\dfrac{x-a}{x-b} = 1 + \dfrac{b-a}{x-b}$

   d) Use c) to plot the function $g(x) = 2\dfrac{x-1}{x-2}$

3) a) Show that $(a+b)^2 = a^2 + 2ab + b^2$

   b) Show that $(a-b)^2 = a^2 - 2ab + b^2$

   c) Show that $(a-b)(a+b) = a^2 - b^2$

   d) Use a) to show the inequality $\dfrac{a+b}{2} \geq \sqrt{ab}$, for $a, b \geq 0$
      When does equality hold?

   e) Use a) to show the inequality $\sqrt{\dfrac{a^2+b^2}{2}} \geq \sqrt{ab}$ for $a, b \geq 0$

   f) Use a) to show the inequality $\sqrt{\dfrac{a^2+b^2}{2}} \geq \dfrac{a+b}{2}$ for $a, b \geq 0$

   g) Let $\max(a,b)$ denote the maximum of the numbers a, b.
      Similarly let $\min(a,b)$ denote the minimum.
      Show that $\max(a,b) \geq \sqrt{\dfrac{a^2+b^2}{2}} \geq \dfrac{a+b}{2} \geq \sqrt{ab} \geq \min(a,b)$ for $a, b \geq 0$