Worksheet 13

1) a) Write down identities for $\sin(\alpha+\beta)$, $\sin(\alpha-\beta)$, $\cos(\alpha+\beta)$, $\cos(\alpha-\beta)$

b) Use your results from a) to show

(i) $2\cos\alpha\cos\beta = \cos(\alpha-\beta) + \cos(\alpha+\beta)$

(ii) $2\sin\alpha\sin\beta = \cos(\alpha-\beta) - \cos(\alpha+\beta)$

(iii) $2\cos\alpha\sin\beta = \sin(\alpha+\beta) + \sin(\alpha-\beta)$

c) Use (b) to show

(i) $\sin\theta \pm \sin\varphi = 2\sin\left(\frac{\theta \pm \varphi}{2}\right)\cos\left(\frac{\theta \mp \varphi}{2}\right)$

(ii) $\cos\theta + \cos\varphi = 2\cos\left(\frac{\theta+\varphi}{2}\right)\cos\left(\frac{\theta-\varphi}{2}\right)$

(iii) $\cos\theta - \cos\varphi = -2\sin\left(\frac{\theta+\varphi}{2}\right)\sin\left(\frac{\theta-\varphi}{2}\right)$

(Hint: Take $\alpha+\beta=\theta$, $\alpha-\beta=\varphi$)

2) Use the addition theorem for sine and cosine to show

$$\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

3) Show that $\sin(3\theta) = -4\sin^3\theta + 3\sin\theta$

4) a) Show that $\tan\theta/2 = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$

5) Use a) to show that $\tan\theta/2 = \frac{\sin\theta}{1+\cos\theta} = \frac{1-\cos\theta}{\sin\theta}$

c) Show $\tan\theta/2 = \csc\theta - \cot\theta$

(See Next Page!)

5) In this problem we define the hyperbolic functions:

$\sinh(x) = \frac{e^x - e^{-x}}{2}$ (hyperbolic sine, read "sinsh")

$\cosh(x) = \frac{e^x + e^{-x}}{2}$ (hyperbolic cosine, read "cosh")

$\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$ (hyperbolic tangent, read "tansh")

a) Sketch the graphs of sinh, cosh and tanh

b) Find an expression for the inverse function of sinh

c) Show that for all x, $\cosh^2(x) - \sinh^2(x) = 1$ (hyperbolic pythagoras)

d) Show that for all x and y,

$$\sinh(x+y) = \sinh(x)\cosh(y) + \cosh(x)\sinh(y)$$
$$\cosh(x+y) = \cosh(x)\cosh(y) + \sinh(x)\sinh(y)$$

(hyperbolic addition theorems)

e) Use d) to find an expression for $\tanh(x+y)$ in terms of $\tanh(x)$ and $\tanh(y)$.

f) Derive the "half-argument formulas" for sinh and cosh

$$\sinh\left(\frac{x}{2}\right) = \sqrt{\frac{1}{2}(\cosh x - 1)} \quad \text{and} \quad \cosh\left(\frac{x}{2}\right) = \sqrt{\frac{1}{2}(\cosh x + 1)}$$

6) a) Compute $\cos(75°)$ using $\cos(75°) = \cos(45° + 30°)$

b) Compute $\cos(75°)$ using $\cos(75°) = \cos(\pi/2 - 15°) = \sin(15°) = \sin\left(\frac{30°}{2}\right)$

c) Show algebraically that your answers for a) and b) coincide.