Worksheet 12

1) a) Find a formula for the area of a regular n-gon P_n inscribed into the unit circle

b) Explain why the area of P_n approximates π for n large

c) Write down an exact expression for the area of P_{12} and P_{24} (Hint: use half-angle formula). How large is $|P_{24} - \pi|$?

d) How large do we need to choose n such that $|P_n - \pi| < 10^{-5}$ (You may use a calculator!)

2) Consider a triangle ABC. Let the angle bisector of angle α intersect side BC at a point D between B and C. Use the law of sines to show

$$\frac{BD}{DC} = \frac{AB}{AC}$$

(Hint: $\sin(180° - \theta) = \sin(\theta)$)

3) Let a, b, c be the sides of a triangle Δ and α, β, γ the angles opposite to those sides

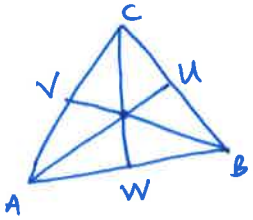
a) Use the law of cosines to show $\sin\gamma = \frac{\sqrt{4a^2b^2 - (a^2+b^2-c^2)^2}}{2ab}$

b) Show that area $\Delta = \frac{1}{4}\sqrt{4a^2b^2 - (a^2+b^2-c^2)^2}$

c) (Bonus Question, worth 1P) Let $s = \frac{a+b+c}{2}$ be the semi perimeter of Δ. Show that area $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

4) a) Let A, B, C be a triangle and let the lines AU, BV, CW intersect in one point, where U, V, W are points on BC, AC and AB respectively. Use the law of sines to show that

$$\frac{AW}{WB} \cdot \frac{BU}{UC} \cdot \frac{CV}{VA} = 1$$



b) What happens if AU, BV and CW are angle bisectors?