Worksheet 11

1) a) Show that the inverse function of an odd function (assuming it exists) is also an odd function.

b) Can an even function have an inverse?

2) Let g be the inverse of the sine function on the interval $[\frac{3\pi}{2}, \frac{5\pi}{2}]$

a) Sketch the graph of g

b) Show $g(x) = \sin^{-1}(x) + 2\pi$

c) What is $g(\cos(\frac{5\pi}{6}))$?

d) What is $\tan(g(\frac{1}{3}))$?

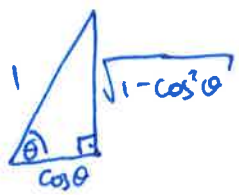
3) a) Sketch the graph of $\cot(x)$ and show $\tan(x) = \cot(\pi/2 - x)$

b) Show that $\cot(x)$ is invertible on $[-\pi/2, 0) \cup (0, \pi/2]$. Call the inverse function $\operatorname{arccot}(x)$ (Arccotangent)

c) Sketch the function $\operatorname{arccot}(x)$

d) (Bonus question, 1P) Explain why $\arctan(x) + \operatorname{arccot}(x) = \begin{cases} \pi/2 & \text{for } x > 0 \\ -\pi/2 & \text{for } x < 0 \end{cases}$

4) Explain why $\sin^{-1}(\sqrt{1 - \cos^2(\theta)}) = \theta$ for $0 \le \theta \le \pi/2$ by looking at the following triangle



What happens when $-\pi/2 \le \theta \le 0$?