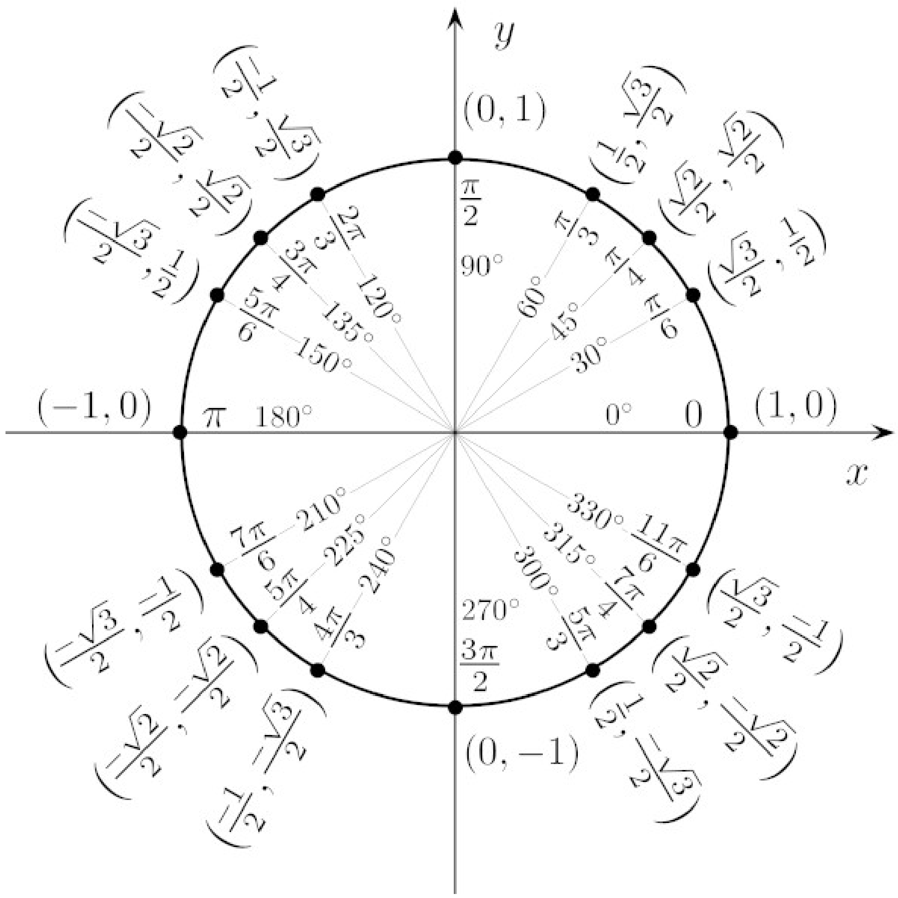
This is a study guide for the Math 32 final, Fall 2013. It is not intended to be exhaustive. The best way to study is to do practice solving problems so I suggest you do as many problems as you can, not just the ones listed here. I would recommend doing the problems in the chapter review sections. The study guide ONLY covers trig, so you will need to review chapters 0-3, sec 7.1-7.2, and Appendix A as well. Good luck!

Section 4.1-4.2

The unit circle: positive angles are measured counter-clockwise, negative angles are measured clockwise. We usually call an angle θ (the Greek letter "theta"), but it can have any name.



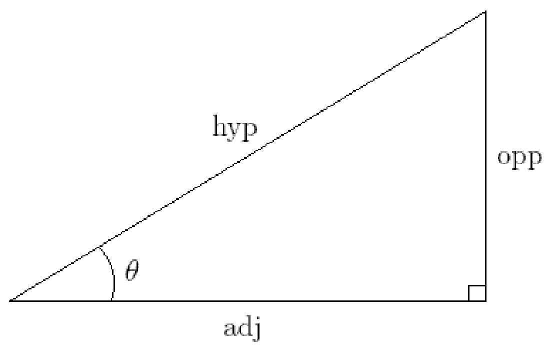
$180^\circ = \pi$ radians.

To convert from radians to degrees, multiply by $\frac{180}{\pi}$.

To convert from degrees to radians, multiply by $\frac{\pi}{180}$.

Section 4.3-4.5

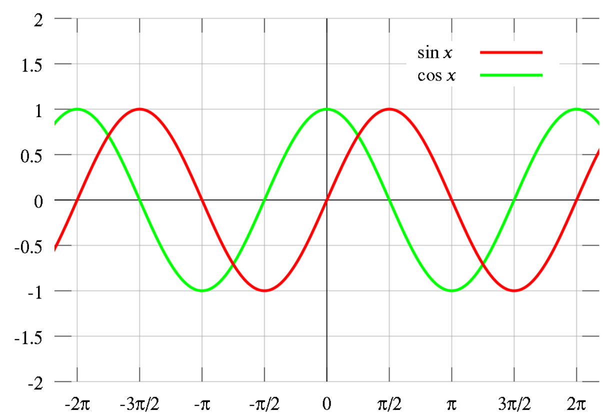
$\cos\theta$ gives us the x coordinate of a point on the unit circle. $\sin\theta$ gives us the y coordinate of a point on the unit circle. $\tan\theta$ gives us the slope of the line that goes from the origin to the point on the unit circle. Alternatively,



$$\cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$\sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{1}{\cos\theta} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{1}{\sin\theta} \qquad \cot\theta = \frac{\text{adj}}{\text{hyp}} = \frac{1}{\tan\theta}$$

Recall that the Domain of a function $f(x)$ is every real number x that the function can accept as in input. The Range of a function $f(x)$ is every real number y that the function can output. Hence $y = f(x)$.
$\sin x$ and $\cos x$ have Domain: $(-\infty, \infty)$ and Range: $[-1, 1]$. When graphed on the x-y coordinate plane, they look like:



Recall that a zero of a function $f(x)$ is a number a such that $f(a) = 0$. Graphically, the function crosses the x-axis at $x = a$.
$\sin x$ has zeros at $x_n = \pm n\pi$ where n is a positive integer including 0. $\sin x = \pm 1$ at $x_n = \pm\frac{(2n-1)\pi}{2}$ where n is a integer that's not 0. You will have to figure out whether it's +1 or -1. Think of the

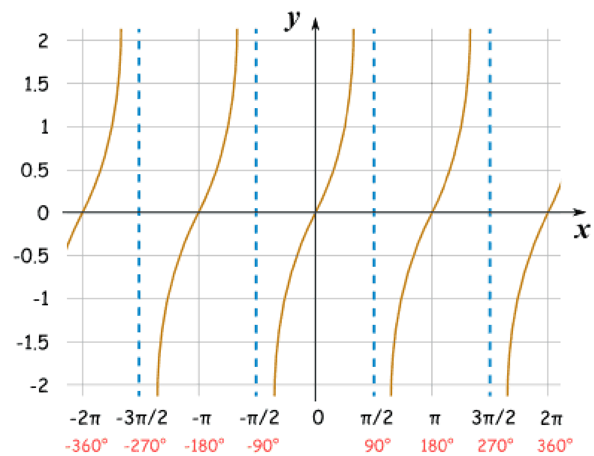
unit circle. When $x = \frac{\pi}{2}$, $\sin x = +1$. When $x = \frac{3\pi}{2}$, $\sin x = -1$. You could follow the pattern: $\sin(\frac{5\pi}{2}) = +1$, $\sin(\frac{7\pi}{2}) = -1$, $\sin(\frac{9\pi}{2}) = +1$, $\sin(\frac{11\pi}{2}) = -1$, etc.

Note that these expressions are both arithmetic sequences. The second one is odd multiples of $\frac{\pi}{2}$ and looks like

$$x_n = \frac{(2n-1)\pi}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots$$

$\cos x$ has zeros at $x_n = \pm\frac{(2n-1)\pi}{2}$ where n is a positive integer that's not 0. $\cos x = \pm 1$ at $x_n = \pm n\pi$ where n is a integer including 0. Note that when $\sin x = \pm 1$, $\cos x = 0$ and when $\cos x = \pm 1$, $\sin x = 0$.

$\tan x = \frac{\sin x}{\cos x}$ so it is undefined when $\cos x = 0$. Remember that $\cos x$ has zeros at $x_n = \pm\frac{(2n-1)\pi}{2}$ where n is a positive integer that's not 0. So that means $\tan x$ is undefined at $x_n = \pm\frac{(2n-1)\pi}{2}$ where n is a positive integer that's not 0.
That means the Domain of $\tan x$ is all real numbers that are not $\pm\frac{(2n-1)\pi}{2}$, where n is a positive integer that's not 0. The Range of $\tan x$ is $(-\infty, \infty)$. It looks like this when graphed in the x-y coordinate plane:



Note that $\tan x$ has vertical asymptotes at $x_n = \pm\frac{(2n-1)\pi}{2}$ where n is a positive integer that's not 0.

Section 4.6, 5.5-5.6: Trig Identities

An identity is an expression that tells us two things are equal. That means we can use them interchangeably. This is useful because one form may be more convenient than the other.

$$\cos^2\theta + \sin^2\theta = 1$$

Remember that $\cos^2\theta$ is the same thing as saying $(\cos\theta)^2$.

$$\cos(-\theta) = \cos(\theta)$$
$$\sin(-\theta) = -\sin(\theta)$$
$$\tan(-\theta) = -\tan(\theta)$$

Recall that we call a function *odd* if $f(-x) = -f(x)$ and *even* if $f(-x) = f(x)$. That means $\sin(x)$ and $\tan(x)$ are odd and $\cos(x)$ is even.

$$\begin{aligned}
\cos(\frac{\pi}{2} - \theta) &= \sin(\theta) \\
\sin(\frac{\pi}{2} - \theta) &= \cos(\theta) \\
\tan(\frac{\pi}{2} - \theta) &= \frac{1}{\tan(\theta)}
\end{aligned}$$

$$\begin{aligned}
\cos(\theta + n\pi) &= \begin{cases} \cos\theta & \text{if } n \text{ is an even integer} \\ -\cos\theta & \text{if } n \text{ is an odd integer} \end{cases} \\
\sin(\theta + n\pi) &= \begin{cases} \sin\theta & \text{if } n \text{ is an even integer} \\ -\sin\theta & \text{if } n \text{ is an odd integer} \end{cases} \\
\tan(\theta + n\pi) &= \tan\theta \quad \text{if } n \text{ is an integer}
\end{aligned}$$

The previous identities you will absolutely have to know for the final. Some of the next ones will be given to you, but not all of them. In particular, the ones with $\tan\theta$ may not be. But that's no problem if you remember $\tan\theta = \frac{\sin\theta}{\cos\theta}$.

$$\cos(2\theta) = 1 - 2\sin^2\theta = 2\cos^2(\theta) - 1 = \cos^2\theta - \sin^2\theta$$

$$\begin{aligned}
\sin(2\theta) &= 2\sin\theta\cos\theta \\
\tan(2\theta) &= \frac{2\tan\theta}{1 - \tan^2\theta}
\end{aligned}$$

$$\begin{aligned}
\cos\frac{\theta}{2} &= \pm\sqrt{\frac{1 + \cos\theta}{2}} \\
\sin\frac{\theta}{2} &= \pm\sqrt{\frac{1 - \cos\theta}{2}} \\
\tan\frac{\theta}{2} &= \frac{1 - \cos\theta}{\sin\theta} = \frac{\sin\theta}{1 + \cos\theta}
\end{aligned}$$

$$\begin{aligned}
\cos(u + v) &= \cos u \cos v - \sin u \sin v \\
\cos(u - v) &= \cos u \cos v + \sin u \sin v \\
\sin(u + v) &= \sin u \cos v + \cos u \sin v \\
\sin(u - v) &= \sin u \cos v - \cos u \sin v
\end{aligned}$$

$$\tan(u + v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

$$\tan(u - v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

You may also not be given $\sin(u - v)$ and $\cos(u - v)$, but you can derive these from $\sin(u + v)$ and $\cos(u + v)$ by replacing v in the identities by $-v$ and remembering that $\cos(-v) = \cos(v)$ and $\sin(-v) = -\sin(v)$.

Section 5.1-5.2

Arccosine: $\cos^{-1} t = \theta$ means $\cos\theta = t$ for $0 \leq \theta \leq \pi$ and $-1 \leq t \leq 1$. The Domain of $\cos^{-1} t$ is $[-1, 1]$ and the Range is $[0, \pi]$.

Arcsine: $\sin^{-1} t = \theta$ means $\sin\theta = t$ for $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ and $-1 \leq t \leq 1$. The Domain of $\sin^{-1} t$ is $[-1, 1]$ and the Range is $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

Arctangent: $\tan^{-1} t = \theta$ means $\tan\theta = t$ for $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ and t is any real number. The Domain of $\tan^{-1} t$ is $(-\infty, \infty)$ and the Range is $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

Recall that the inverse of $f(x)$, we call $f^{-1}(x)$, is defined such that $f(f^{-1}(x)) = f^{-1}(f(x)) = x$. That means

$$\begin{aligned}
\cos^{-1}(\cos(\theta)) &= \theta \quad \text{for every } \theta \text{ in } [0, \pi] \\
\sin^{-1}(\sin(\theta)) &= \theta \quad \text{for every } \theta \text{ in } [-\frac{\pi}{2}, \frac{\pi}{2}] \\
\tan^{-1}(\tan(\theta)) &= \theta \quad \text{for every } \theta \text{ in } [-\frac{\pi}{2}, \frac{\pi}{2}]
\end{aligned}$$

And

$$\begin{aligned}
\cos(\cos^{-1}(t)) &= t \quad \text{for every } t \text{ in } [-1, 1] \\
\sin(\sin^{-1}(t)) &= t \quad \text{for every } t \text{ in } [-1, 1] \\
\tan(\tan^{-1}(t)) &= t \quad \text{for every real number } t
\end{aligned}$$

We also have the following identities

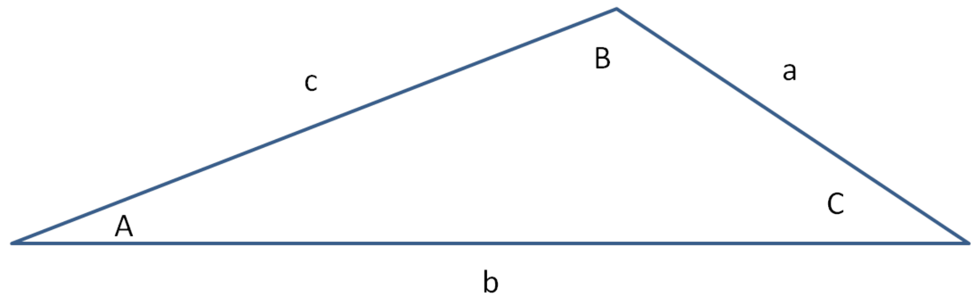
$$\begin{aligned}
\cos^{-1}(-t) &= \pi - \cos^{-1}(t) \\
\sin^{-1}(-t) &= -\sin^{-1}(t) \\
\tan^{-1}(-t) &= -\tan^{-1}(t)
\end{aligned}$$

$$\cos^{-1} t + \sin^{-1} t = \frac{\pi}{2}$$

Remember $-1 \leq t \leq 1$ for $\cos^{-1} t$ and $\sin^{-1} t$. t can be any real number for $\tan^{-1} t$.

Section 5.3-5.5

In this section, consider the following triangle



Recall the area of a triangle $Area = \frac{1}{2}(base \times height)$. With some trig, we can also say

$$Area = \frac{1}{2}bc\sin A = \frac{1}{2}ab\sin C = \frac{1}{2}ac\sin B$$

Law of Sines:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of Cosines:

$$\begin{aligned} c^2 &= a^2 + b^2 - 2ab\cos C \\ b^2 &= a^2 + c^2 - 2ac\cos B \\ a^2 &= c^2 + b^2 - 2cb\cos A \end{aligned}$$

Suppose you have an angle θ that is small. Then the following *approximate* expressions may be useful:

$$\begin{aligned} \sin\theta &\approx \theta \\ \tan\theta &\approx \theta \\ \cos\theta &\approx 1 - \frac{\theta^2}{2} \\ \frac{\sin\theta}{\theta} &\approx 1 \end{aligned}$$

Here θ *has to be in radians*. The meaning of a "small" angle is a little bit subjective. Typically it will be much less than the number 1. A good rule of thumb is ~ 0.1.

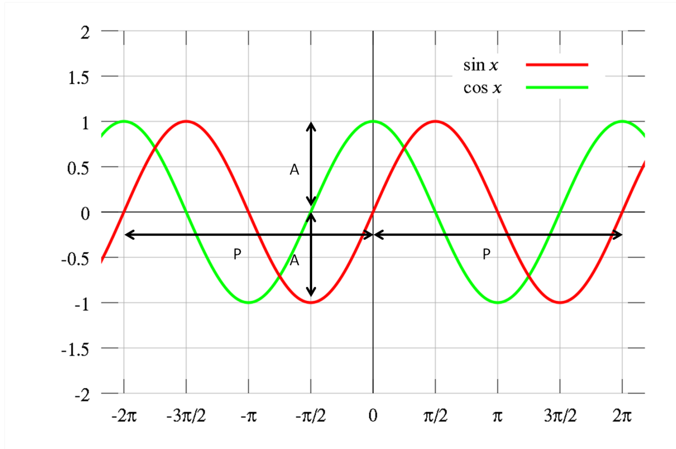
Section 6.1

The Amplitude of a sine or cosine wave is half the distance between the maximum and the minimum:

$$A = \frac{1}{2}(\text{Max} - \text{Min})$$

A quick way to find the amplitude of a sine or cosine function is to look at the number multiplying the sine or cosine. Example: $3\sin x$ has amplitude 3.

A function is called periodic if $f(x+P) = f(x)$. We call the period of the function P. That means the function will repeat itself every P units on the x-axis. $\sin x$ and $\cos x$ both have $P = 2\pi$ and $\tan x$ has $P = \pi$.



Recall that if we multiply a function by a constant like this: $g(x) = cf(x)$, we stretch it *vertically* in the x-y plane. If $c > 1$, the graph gets bigger and if $0 < c < 1$ the graph gets smaller. If $c < 0$, it will mirror itself about the x-axis. If we stretch a trig function vertically, we change its amplitude.

Recall that if we add (or subtract) a constant to a function like this: $g(x) = f(x) + c$, it shifts vertically by c units. If we add a positive number (e.g. $f(x) + 2$), it moves up. If we subtract a positive number (e.g. $f(x) - 2$), it moves down.

Recall that if we multiply the input of a function by a constant like this: $g(x) = f(cx)$, we stretch it *horizontally* in the x-y plane. If $c > 1$, the graph gets smaller and if $0 < c < 1$ the graph gets bigger. If $c < 0$, it will mirror itself about the y-axis. If we stretch a trig function horizontally, we change its period. A convenient way to change the period of a sine or cosine wave is to write it like this

$$\sin\Big(\frac{2\pi}{P}x\Big) \qquad \cos\Big(\frac{2\pi}{P}x\Big)$$

Then we just pick P to be the period we want. For example, if we want $P = 2\pi$, we would get

$$\sin\Big(\frac{2\pi}{P}x\Big) = \sin\Big(\frac{2\pi}{2\pi}x\Big) = \sin(x)$$

which makes sense since we know the period of $\sin(x)$ is 2π. If someone gives us some random sine wave and wants us to find the period, we just set $\frac{2\pi}{P}$ equal to whatever is multiplying x and solve for P.

Example: Find the period of $\cos(0.37x - 2)$.

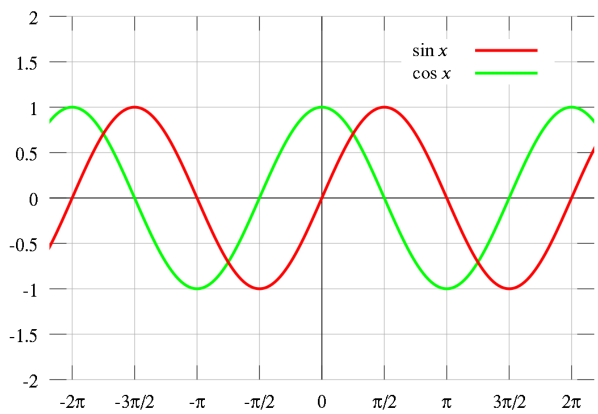
$$\frac{2\pi}{P} = 0.37 \implies P = \frac{2\pi}{0.37}$$

The period of $\cos(0.37x - 2)$ is $\frac{2\pi}{0.37} \approx \frac{6.28}{0.37} \approx 20$. Notice how subtracting 2 inside cos didn't affect our calculation That means $\cos(0.37x - 2)$ and $\cos(0.37x)$ have the same period. *The only thing that will change the period of a trig function is stretching it horizontally, i.e., $f(cx)$.*

Phase Shift

Recall that if we add (or subtract) a constant to the input of a function like this: $g(x) = f(x + c)$, it shifts horizontally by c units. If we add a positive number (e.g. $f(x + 2)$), it moves to the left. If we subtract a positive number (e.g. $f(x - 2)$), it moves to the right. For trig functions, we call this a *phase shift*.

As an example look at the following graph:



We observe that $\cos(x)$ is just $\sin(x)$ shifted to the *left* by $\frac{\pi}{2}$. So $\cos(x) = \sin(x + \frac{\pi}{2})$. We can also look at it this way: $\sin(x)$ is just $\cos(x)$ shifted to the *right* by $\frac{\pi}{2}$. So $\sin(x) = \cos(x - \frac{\pi}{2})$.