DO NOT TURN OVER UNTIL INSTRUCTED TO DO SO.

NO CALCULATORS PERMITTED.

EXAM TIME IS 170 MINUTES.

THE EXAM CONSISTS OF 10 QUESTIONS.

Your name: ________________________________

Your SID: ________________________________

Your Section and GSI: ________________________________

Question 1	/ 20
Question 2	/ 20
Question 3	/ 20
Question 4	/ 20
Question 5	/ 20
Question 6	/ 20
Question 7	/ 20
Question 8	/ 20
Question 9	/ 20
Question 10	/ 20
Total	/ 200

1. Evaluate the following expressions

   (a) $\sin(135°)$

   (b) $\cos(\frac{5\pi}{3})$

   (c) $\cot(\frac{\pi}{4})$

   (d) $\sin^{-1}(\sin(\frac{7\pi}{8}))$

   (e) $\cos(\tan^{-1}(\frac{1}{3}))$

2. (a) Derive the double-angle formula for cosine from the addition theorems.
   (b) Use (a) to derive the half-angle formulae for sine and cosine.
   (c) Compute $\sin(\frac{45^\circ}{2})$.

3. Let A, B, C be the angles of a triangle and a, b, c the lengths of the opposite sides respectively.

   (a) Write down the law of sines and explain how to derive it.

   (b) Assuming $a = 5$, $A = \frac{\pi}{6}$ and $B = \frac{\pi}{8}$ find the other angles and sidelengths of the triangle. (Note: You may use your result from Question 2)

4. (a) Let $f(x) = \frac{e^x - e^{-x}}{2}$. Sketch the graph of $f(x)$.
   (b) Use the graph to explain why $f(x)$ is invertible.
   (c) Is $f(x)$ an even or odd function?
   (d) Find the inverse of $f(x)$.

5. (a) Sketch the graphs of $\sin(x)$, $\cos(x)$ and $\tan(x)$.
   (b) How many zeros does $\sin(x)$ have in the interval $(\pi, \frac{27\pi}{2}]$?
   (c) Find the amplitude and the period of the function $f(x) = \frac{1}{2} + \frac{1}{2}\sin(\frac{\pi}{2}x - \pi)$.
   (d) Sketch the function $f(x)$.

(a) Show that $\tan^2(\theta) + 1 = \sec^2(\theta)$

(b) Show using the addition theorem for sine and cosine that

$$\tan(\theta_1 + \theta_2) = \frac{\tan(\theta_1) + \tan(\theta_2)}{1 - \tan(\theta_1)\tan(\theta_2)}$$

.

(c) Use b) to find an expression for $\tan(\theta_1 - \theta_2)$.

6. (a) Write down the equation for a circle K with centre $C : (2, 3)$ and radius 2.
   (b) Write down the equation of the line l that passes through C and has gradient $-\sqrt{3}$.
   (c) Find the point P of interception of the line l with circle K, for which the y-coordinate is less than 3.
   (d) At what angle does the line l intersect the x-axis?

7. (a) Derive the base change formula for logarithms

   (b) Compute $\log_{27}(81)$ and $e^{\frac{2}{3}\ln(8)}$.

   (c) Solve $4\log_{16}(x) + 3\log_8(x) + 2\log_2(x) = 4$.

8. (a) Sketch the graph of $f(x) = |x+2| - |x-2|$

   (b) Find the area under the graph of $f(x)$ from $x = 1$ to $x = 5$

   (c) Find all x for which $2 \leq ||x+2| - |x-2|| \leq 3$

9. (a) Define what a function is.

   (b) Define the terms injective and surjective for functions.

   (c) For the function $f(x) = x^2$ find the pre-image of $(2, 4] \cup [5, 25)$. Write your answer as a union of intervals.

10. Let $f(x) = \frac{x^3+x}{x^2+2x+1}$.

    (a) Find the zeros and vertical asymptotes of $f(x)$. Justify your answer.

    (b) What is the behaviour of $f(x)$ near $\pm\infty$?

    (c) Where is $f(x)$ positive? Express your answer as an interval.

    (d) Sketch the graph of $f(x)$.

END OF EXAM

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