DO NOT TURN OVER UNTIL INSTRUCTED TO DO SO.

NO CALCULATORS PERMITTED.

EXAM TIME IS 60 MINUTES.

THE EXAM CONSISTS OF 5 QUESTIONS.

Your name: ______________________________

Your SID: ______________________________

Your Section and GSI: ______________________________

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Total	/ 100

1. Consider the rational function

$$f(x) = \frac{(x-1)(x+1)(x+2)}{x^3+x}$$

(a) Does $f(x)$ have a horizontal asymptote? If so, what is it?

(b) Does $f(x)$ have vertical asymptotes? If so, what are they?

(c) Does $f(x)$ have any x-intercepts? If so, what are they?

(d) Does $f(x)$ have a y-intercept? If so, what is it?

(e) Find the region in which $f(x)$ is positive. Express your answer as a union of intervals.

(a) $f(x) \approx \frac{x^3}{x^3} = 1$ near $\pm\infty$ $\Rightarrow$ horizontal asymptote $y=0$

(b) $x^3+x = x(x^2+1)$ $\Rightarrow$ vertical asymptote at $x=0$

(c) x-intercepts where $f(x)=0 \iff (x-1)(x+1)(x+2)=0$

$\iff x = 1, -1$ or -2

(d) $f(x)$ does not have a y-intercept ($f(x)$ not defined at $x=0$)

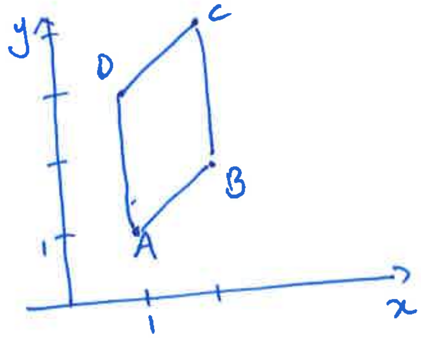
(e)

		-2		-1		0		1	
$x-1$	$-$		$-$		$-$		$-$		$+$
$x+1$	$-$		$-$		$+$		$+$		$+$
$x+2$	$-$		$+$		$+$		$+$		$+$
x	$-$		$-$		$-$		$+$		$+$
x^2+1	$+$		$+$		$+$		$+$		$+$
$f(x)$	$+$		$-$		$+$		$-$		$+$

So $f(x)$ positive on $(-\infty,-2) \cup (-1,0) \cup (1,\infty)$

2. Let $A : (1,1)$, $B : (2,2)$, $C : (2,4)$ and $D : (1,3)$
   (a) Sketch the parallelogram ABCD in a coordinate system
   (b) Find the area of ABCD
   (c) Find the perimeter of ABCD
   (d) Find the midpoint of ABCD (i.e. the intersection of AC and DB)

(a)



(b) height $= 1$ (distance of lines AD and BC)

length $AD = 2$

$\Rightarrow$ Area $= 2$

(c) length $AB = \sqrt{1^2+1^2} = \sqrt{2}$

$\Rightarrow$ perimeter $= 2(AD + AB) = 2(2+\sqrt{2}) = 4+2\sqrt{2}$

(d) midpoint $=$ midpoint of line section AC

$= \left(\frac{1+2}{2}, \frac{1+4}{2}\right) = (1.5, 2.5)$

3. The following equation describes an ellipse

$$4x^2 + 9y^2 - 8x + 36y + 4 = 0$$

(a) Write the above equation in the form $\left(\frac{x-h}{a}\right)^2 + \left(\frac{y-v}{b}\right)^2 = 1$

(b) Find the area of the ellipse

(c) Find the center of the ellipse

(a) $4x^2 - 8x + 9y^2 + 36y + 4 = 0$

$\Leftrightarrow 4(x^2 - 2x + 1 - 1) + 9(y^2 + 4y + 4 - 4) + 4 = 0$

$\Leftrightarrow 4[(x-1)^2 - 1] + 9((y+2)^2 - 4) + 4 = 0$

$\Leftrightarrow 4(x-1)^2 + 9(y+2)^2 = 36$

$\Leftrightarrow \frac{(x-1)^2}{9} + \frac{(y+2)^2}{4} = 1$

$\Leftrightarrow \left(\frac{x-1}{3}\right)^2 + \left(\frac{y+2}{2}\right)^2 = 1$

(b) Centre: $(1, -2)$

(c) area: $2 \cdot 3 \cdot \pi = 6\pi$

4. This is a multiple choice question. You do not need to show any work but you will lose two points for any wrong answer.

   (a) Which of the following numbers is greatest? (Hint: $2^{10} = 1024$)

   i) 2^{100} ii) 10^{30} iii) 9^{29} iv) 2^{98}

   (b) $\dfrac{2^{\left(2^{(2^2)}\right)}}{(2^2)^{(2^2)}}$ is equal to

   i) 128 ii) 256 iii) 512 iv) 1024

   (c) $2^{4 \log_{16} 2 - 1}$ is equal to

   i) $\frac{1}{2}$ ii) 1 iii) 2 iv) 4

   (d) area$(\frac{1}{x}, 3, e^2)$ is equal to

   i) $2 - \ln(3)$ ii) $\ln(3) - 2$ iii) $e^2 - 3$ iv) $3 - e^2$

   (e) $\ln\left(\frac{x}{x+1}\right) + \ln(2x+2) = 1$ has solution

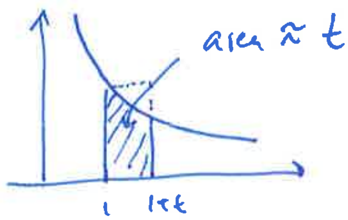
   i) e ii) $2e$ iii) $\frac{e}{2}$ iv) 1

5. (a) Explain why for t small we have $\ln(1+t) \approx t$
   (b) Use part (a) to show that for t small $\exp(t) \approx 1+t$
   (c) Use part (b) to show that for n large $\exp(t) \approx \left(1+\frac{t}{n}\right)^n$
   (d) Use part (a) to find an approximate value of

$$\frac{e^{1.001}e^{2.002}}{e^3}$$

(a) $\ln(1+t) = \text{area}\,(1/x, 1, 1+t)$

Sketch



Thus $\ln(1+t) \approx t$ for small t

(b) Exponentiating we obtain $e^{\ln(1+t)} \approx e^t$ for small t

$\Rightarrow \quad e^t \approx 1+t$ for small t

(c) We have $e^t = \left(e^{t/n}\right)^n \approx \left(1+t/n\right)^n$, where n is chosen such that t/n very small, and $e^{t/n} \approx 1+t/n$ is a good approximation

(d) $\dfrac{e^{1.001}e^{2.002}}{e^3} = \dfrac{e^1 e^2 e^{0.001} e^{0.002}}{e^3} = e^{0.003}$

$\approx 1 + 0.003$

≈ 1.003

END OF EXAM

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