

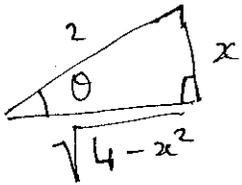
Homework 2 : Solutions

Section 7.3

$$-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$$

Q1 // $x = 2 \sin \theta \Rightarrow \frac{dx}{d\theta} = 2 \cos \theta \Rightarrow dx = 2 \cos \theta d\theta$

$$\Rightarrow \int \frac{dx}{x^2 \sqrt{4-x^2}} = \int \frac{2 \cos \theta d\theta}{4 \sin^2 \theta 2 \cos \theta} = \frac{1}{4} \int \csc^2 \theta d\theta = -\frac{1}{4} \cot \theta + C$$



$$\Rightarrow \int \frac{dx}{x^2 \sqrt{4-x^2}} = -\frac{1}{4} \cdot \frac{\sqrt{4-x^2}}{x} + C$$

Q4 // Substitute $x = \sin \theta$ ($-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$) \Rightarrow

$$\frac{dx}{d\theta} = \cos \theta \Rightarrow dx = \cos \theta d\theta \Rightarrow$$

$$\int_0^1 x^3 \sqrt{1-x^2} dx = \int_{\arcsin(0)}^{\arcsin(1)} \sin^3 \theta \cos^2 \theta d\theta$$

$$= \int_0^{\frac{\pi}{2}} \sin^3 \theta \cos^2 \theta d\theta$$

Substitute $u = \cos \theta \Rightarrow \frac{du}{d\theta} = -\sin \theta$

$$\Rightarrow d\theta = \frac{1}{-\sin \theta} du \Rightarrow \int_0^{\frac{\pi}{2}} \sin^3 \theta \cos^2 \theta d\theta = - \int_1^0 \sin^2 \theta u^2 du$$

$$= \int_0^1 (1-u^2) u^2 du = \left[\frac{1}{3} u^3 - \frac{1}{5} u^5 \right]_0^1 = \frac{1}{3} - \frac{1}{5} = \frac{2}{15} //$$

Substitute

$$x = a \tan \theta \Rightarrow \left(-\frac{\pi}{2} < \theta < \frac{\pi}{2}\right)$$

$$\frac{dx}{d\theta} = a \sec^2 \theta \Rightarrow dx = a \sec^2 \theta d\theta$$

$$\begin{aligned} \Rightarrow \int_0^a \frac{dx}{(a^2 + x^2)^{3/2}} &= \int_0^{\frac{\pi}{4}} \frac{a \sec^2 \theta}{a^3 \sec^3 \theta} d\theta \\ &= \frac{1}{a^2} \int_0^{\frac{\pi}{4}} \cos \theta d\theta = \frac{1}{a^2} \left[\sin \theta \right]_0^{\frac{\pi}{4}} \\ &= \frac{1}{a^2} \sin \frac{\pi}{4} = \frac{1}{\sqrt{2} a^2} // \end{aligned}$$

Q15 //

Substitute

$$x = a \sin \theta \Rightarrow \left(-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}\right)$$

$$\frac{dx}{d\theta} = a \cos \theta$$

$$\Rightarrow dx = a \cos \theta d\theta$$

$$\begin{aligned} \Rightarrow \int_0^a x^2 \sqrt{a^2 - x^2} dx &= \int_0^{\frac{\pi}{2}} a^2 \sin^2 \theta \cos^2 \theta d\theta \\ &= \frac{a^2}{4} \int_0^{\frac{\pi}{2}} (2 \sin \theta \cos \theta)^2 d\theta \\ &= \frac{a^2}{4} \int_0^{\frac{\pi}{2}} (\sin 2\theta)^2 d\theta \\ &= \frac{a^2}{4} \int_0^{\frac{\pi}{2}} \left(\frac{1 - \cos(4\theta)}{2} \right) d\theta \\ &= \frac{a^2}{8} \left[\theta - \frac{1}{4} \sin(4\theta) \right]_0^{\frac{\pi}{2}} \\ &= \frac{\pi a^2}{16} // \end{aligned}$$

Q25 // Complete the square :

$$x^2 + x + 1 = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$$

Do a substitution $x + \frac{1}{2} = \frac{\sqrt{3}}{2} \tan \theta \Rightarrow$

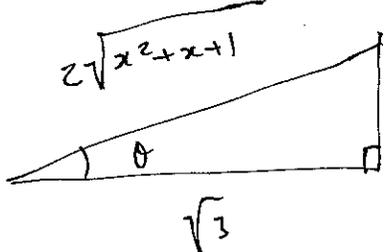
$$x = \frac{\sqrt{3}}{2} \tan \theta - \frac{1}{2} \Rightarrow \frac{dx}{d\theta} = \frac{\sqrt{3}}{2} \sec^2 \theta \Rightarrow$$

$$dx = \frac{\sqrt{3}}{2} \sec^2 \theta d\theta. \text{ Hence}$$

$$\int \frac{x}{\sqrt{x^2+x+1}} dx = \int \frac{\left(\frac{\sqrt{3}}{2} \tan \theta - \frac{1}{2}\right) \frac{\sqrt{3} \sec^2 \theta d\theta}{\frac{\sqrt{3}}{2} \sec \theta}}{\frac{\sqrt{3}}{2} \sec \theta} d\theta$$

$$= \int \frac{\sqrt{3}}{2} \tan \theta \sec \theta d\theta - \int \frac{1}{2} \sec \theta d\theta$$

$$= \frac{\sqrt{3}}{2} \sec \theta - \frac{1}{2} \ln |\sec \theta + \tan \theta| + C$$

 $\Rightarrow \sec \theta = \frac{2}{\sqrt{3}} \sqrt{x^2+x+1}$
 $\tan \theta = \frac{2}{\sqrt{3}} \left(x + \frac{1}{2}\right)$

$$= \sqrt{x^2+x+1} + \frac{1}{2} \ln \left| \frac{2}{\sqrt{3}} \left(\sqrt{x^2+x+1} + x + \frac{1}{2}\right) \right| + C$$

$$= \sqrt{x^2+x+1} - \frac{1}{2} \ln \left| \sqrt{x^2+x+1} + \left(x + \frac{1}{2}\right) \right| + C$$

(Remember that $\ln(ab) = \ln(a) + \ln(b)$)

easier.

$$\frac{dx}{dt} = -r \sin(t) \Rightarrow dx = -r \sin(t) dt \Rightarrow$$

$$\int_{r \cos \theta}^r \sqrt{r^2 - x^2} dx = - \int_{\theta}^0 r^2 \sin^2 t dt = r^2 \int_0^{\theta} \sin^2 t dt$$

$$= r^2 \int_0^{\theta} \frac{1 - \cos 2t}{2} dt = \frac{r^2}{2} \left[t - \frac{1}{2} \sin 2t \right]_0^{\theta}$$
$$= \frac{\theta r^2}{2} - \frac{r^2}{4} \sin 2\theta$$

$$\text{Area of sector} = \underbrace{\frac{1}{2} (r \cos \theta) (r \sin \theta)}_{\text{Area of right angle triangle}} + \frac{\theta r^2}{2} - \frac{r^2}{4} \sin 2\theta$$

$$= \frac{r^2}{4} \sin 2\theta + \frac{\theta r^2}{2} - \frac{r^2}{4} \sin 2\theta = \frac{\theta r^2}{2} //$$

Q43 Model the big arch as the function

$$y = \sqrt{R^2 - x^2} - \sqrt{R^2 - r^2}$$

$$\text{Area of Curve} = \frac{1}{2} \pi r^2 - \int_{-r}^r (\sqrt{R^2 - x^2} - \sqrt{R^2 - r^2}) dx$$

$$= \frac{1}{2} \pi r^2 + 2r \sqrt{R^2 - r^2} - \int_{-r}^r \sqrt{R^2 - x^2} dx$$

substitution $x = R \sin \theta$ ($\frac{r}{2} = \dots$)

$$\frac{dx}{d\theta} = R \cos \theta \Rightarrow dx = R \cos \theta d\theta \Rightarrow$$

$$\int_{-r}^r \sqrt{R^2 - x^2} dx = \int_{-\arcsin(\frac{r}{R})}^{\arcsin(\frac{r}{R})} R^2 \cos^2 \theta d\theta$$

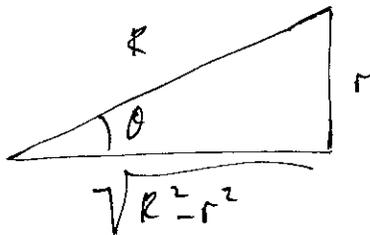
$$= R^2 \int_{-\arcsin(\frac{r}{R})}^{\arcsin(\frac{r}{R})} \frac{1 + \cos 2\theta}{2} d\theta$$

$$= \frac{R^2}{2} \left[\theta + \frac{1}{2} \sin 2\theta \right]_{-\arcsin(\frac{r}{R})}^{\arcsin(\frac{r}{R})}$$

$$= R^2 \arcsin\left(\frac{r}{R}\right) + \frac{R^2}{2} \sin\left(2 \arcsin\left(\frac{r}{R}\right)\right)$$

$$\text{Area of lune} = \frac{1}{2} \pi r^2 + 2r \sqrt{R^2 - r^2} - R^2 \arcsin\left(\frac{r}{R}\right)$$

$$+ \frac{-R^2}{2} \cdot 2 \cdot \frac{r}{R} \cdot \frac{\sqrt{R^2 - r^2}}{R}$$



$$= \frac{1}{2} \pi r^2 + r \sqrt{R^2 - r^2} - R^2 \arcsin\left(\frac{r}{R}\right)$$

Q7// change the variable $u = x - 1 \Rightarrow \frac{du}{dx} = 1 \Rightarrow du = dx$

$$\begin{aligned}\Rightarrow \int \frac{x^4}{(x-1)} dx &= \int \frac{(u+1)^4}{u} du \\ &= \int (u^3 + 4u^2 + 6u + 4 + u^{-1}) du \\ &= \frac{1}{4} u^4 + \frac{4}{3} u^3 + 3u^2 + 4u + \ln|u| + C \\ &= \frac{1}{4} (x-1)^4 + \frac{4}{3} (x-1)^3 + 3(x-1)^2 + 4(x-1) \\ &\quad + \ln|(x-1)| + C\end{aligned}$$

Q10//

$$\begin{aligned}\frac{y}{(y+4)(2y-1)} &= \frac{A}{y+4} + \frac{B}{2y-1} \\ &= \frac{A(2y-1) + B(y+4)}{(y+4)(2y-1)} = \frac{(2A+B)y + (4B-A)}{(y+4)(2y-1)}\end{aligned}$$

$$\begin{aligned}\Rightarrow \begin{cases} 2A + B = 1 \\ 4B - A = 0 \end{cases} &\Rightarrow A = 4B \Rightarrow 9B = 1 \Rightarrow B = \frac{1}{9}, A = \frac{4}{9}\end{aligned}$$

$$\begin{aligned}\int \frac{y}{(y+4)(2y-1)} &= \int \frac{\left(\frac{4}{9}\right)}{(y+4)} + \frac{\left(\frac{1}{9}\right)}{(2y-1)} dy \\ &= \frac{4}{9} \ln|y+4| + \frac{1}{18} \ln|2y-1| + C\end{aligned}$$

$$\frac{x^2 - 2x - 1}{(x-1)^2(x^2+1)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{Cx+D}{x^2+1}$$

$$= \frac{A(x-1)(x^2+1) + B(x^2+1) + (Cx+D)(x-1)^2}{(x-1)^2(x^2+1)}$$

$$= \frac{(A+C)x^3 + (-A+B-2C+D)x^2 + (A+C-2D)x + (-A+B+D)}{(x-1)^2(x^2+1)}$$

$$\Rightarrow A+C=0$$

$$-A+B-2C+D=1$$

$$A+C-2D=-2$$

$$-A+B+D=-1$$

$$\Rightarrow A=-C \Rightarrow A+B+D=1$$

$$-2D=-2$$

$$-A+B+D=-1$$

$$\Rightarrow D=1$$

$$A+B=0$$

$$-A+B=-2$$

$$\Rightarrow A=1$$

$$B=-1$$

$$C=-1$$

$$D=1$$

$$= \int \frac{x^2 - 2x - 1}{(x-1)^2(x^2+1)} dx = \int \left(\frac{1}{(x-1)} - \frac{1}{(x-1)^2} + \frac{-x+1}{x^2+1} \right) dx$$

$$= \ln |(x-1)| + \frac{1}{(x-1)} - \frac{1}{2} \ln |x^2+1| + \arctan(x) + C$$

35 //

$$\frac{1}{x(x^2+4)^2}$$

$$= \frac{A}{x} + \frac{Bx+C}{x^2+4} + \frac{Dx+E}{(x^2+4)^2}$$

$$= \frac{A(x^2+4)^2 + (Bx+C)(x)(x^2+4) + (Dx+E)x}{x(x^2+4)^2}$$

$$= \frac{(A+B)x^4 + (2Ax^3 + 4Bx^2 + Dx^2 + 4Cx + E)x + 16A}{x(x^2+4)^2}$$

$$x(x^2+4)^2$$

$$\Rightarrow A+B=0$$

$$C=0$$

$$2A+4B+D=0$$

$$4C+E=0$$

$$16A=1$$

$$\Rightarrow A = \frac{1}{16}, \quad B = -\frac{1}{16}, \quad C=0$$

$$\Rightarrow D = -\frac{1}{4}, \quad E=0$$

$$\Rightarrow \int \frac{dx}{x(x^2+4)^2}$$

$$= \frac{1}{16} \int \frac{1}{x} dx + \frac{-1}{16} \int \frac{x}{x^2+4} dx$$

$$- \frac{1}{4} \int \frac{x}{(x^2+4)^2} dx$$

$$= \frac{1}{16} \ln|x| - \frac{1}{32} \ln|x^2+4|$$

$$+ \frac{1}{8} \cdot \frac{1}{x^2+4} + C //$$

Q44 // substitution $u = \sqrt{x+1}$ $\Rightarrow \frac{du}{dx} = \frac{1}{2}$ $\Rightarrow dx = 2u du$

$$\begin{aligned} \Rightarrow \int \frac{\sqrt{x+1}}{x} dx &= \int \frac{2u^2}{u^2-1} du \\ &= \int \left(2 + \frac{2}{u^2-1} \right) du \\ &= \int \left(2 + \frac{1}{u-1} - \frac{1}{u+1} \right) du \\ &= 2u + |u|u-1| - |u|u+1| + C \\ &= 2\sqrt{x+1} + |u|\sqrt{x+1}-1| - |u|\sqrt{x+1}+1| + C \end{aligned}$$

Q45 // $u = \sqrt[6]{x}$ $\frac{du}{dx} = \frac{1}{6} x^{-\frac{5}{6}} \Rightarrow dx = 6x^{\frac{5}{6}} du = 6u^5 du$

$$\begin{aligned} \Rightarrow \int \frac{1}{\sqrt{x} - \sqrt[3]{x}} dx &= \int \frac{6u^5}{u^3 - u^2} du \\ &= 6 \int \frac{u^3}{u-1} du \end{aligned}$$

(Now substitute $t = u - 1 \Rightarrow du = dt$)

$$= 6 \int \frac{(t+1)^3}{t} dt = 6 \int \left[t^2 + 3t + 3 + \frac{1}{t} \right] dt$$

$$= 2t^3 + 9t^2 + 18t + 6\ln|t| + C$$

$$= 2(\sqrt[6]{x} - 1)^3 + 9(\sqrt[6]{x} - 1)^2 + 18(\sqrt[6]{x} - 1) + 6\ln|\sqrt[6]{x} - 1| + C$$

Q46 Substitution $y = \sqrt{1+\sqrt{x}} \Rightarrow x = (1-y^2)^2$

and $dx = -4(1-y^2)y dy$. Hence

$$\int \frac{\sqrt{1+\sqrt{x}}}{x} dx = 4 \int \frac{-y}{(1-y^2)^2} (1-y^2)y dy$$

$$= 4 \int \frac{-y^2}{(1-y^2)} dy$$

$$= 4 \int \left(1 - \frac{1}{1-y^2}\right) dy$$

$$= 4y - 4 \int \frac{1}{1-y^2} dy$$

$$\frac{1}{1-y^2} = \frac{A}{1-y} + \frac{B}{1+y} \Rightarrow 1 = A(1+y) + B(1-y)$$

$$\Rightarrow \begin{aligned} A - B &= 0 \\ A + B &= 1 \end{aligned} \Rightarrow A = B = \frac{1}{2}$$

$$\Rightarrow \int \frac{1}{1-y^2} dy = \frac{1}{2} \int \frac{1}{1-y} dy + \frac{1}{2} \int \frac{1}{1+y} dy$$

$$= \frac{1}{2} \ln|1+y| - \frac{1}{2} \ln|1-y| + C$$

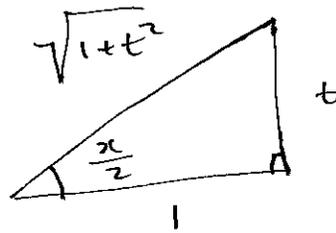
Hence $\int \frac{\sqrt{1+\sqrt{x}}}{x} dx = 4\sqrt{1+\sqrt{x}} + 2\ln|1-\sqrt{1+\sqrt{x}}| - 2\ln|1+\sqrt{1+\sqrt{x}}| + C$

$$\int x \arctan(x) dx \quad \begin{matrix} f(x) \\ g'(x) \end{matrix} \quad \text{Choose } g(x) = \frac{1}{2}x^2$$

$$\begin{aligned} \Rightarrow \int x \arctan(x) dx &= \frac{1}{2}x^2 \arctan(x) - \frac{1}{2} \int x^2 \left(\frac{1}{x^2+1} \right) dx \\ &= \frac{1}{2}x^2 \arctan(x) - \frac{1}{2} \int \left(1 - \frac{1}{x^2+1} \right) dx \\ &= \frac{1}{2}x^2 \arctan(x) - \frac{1}{2}x + \frac{1}{2} \arctan(x) + C \end{aligned}$$

Q59

a) $t = \tan\left(\frac{x}{2}\right)$



$$\cos\left(\frac{x}{2}\right) = \frac{1}{\sqrt{1+t^2}}$$

$$\sin\left(\frac{x}{2}\right) = \frac{t}{\sqrt{1+t^2}}$$

$$\begin{aligned} \text{b) } \cos(x) &= \cos\left(\frac{x}{2} + \frac{x}{2}\right) = \cos^2\left(\frac{x}{2}\right) - \sin^2\left(\frac{x}{2}\right) \\ &= \frac{1}{1+t^2} - \frac{t^2}{1+t^2} \\ &= \frac{1-t^2}{1+t^2} \end{aligned}$$

$$\begin{aligned} \sin(x) &= \sin\left(\frac{x}{2} + \frac{x}{2}\right) = 2 \sin\left(\frac{x}{2}\right) \cos\left(\frac{x}{2}\right) \\ &= \frac{2t}{1+t^2} \end{aligned}$$

$$\text{c) } \frac{dt}{dx} = \frac{1}{2} \sec^2\left(\frac{x}{2}\right) = \frac{1}{2 \cos^2\left(\frac{x}{2}\right)} = \frac{1+t^2}{2} \Rightarrow dx = \frac{2}{1+t^2} dt$$

section 7.5

Q3 //
$$\int \frac{\sin(x) + \sec(x)}{\tan(x)} dx = \int \csc(x) + \csc(x) dx$$

$$= \sin(x) + \ln | \csc(x) - \cot(x) | + C$$

Q8 //
$$\int \frac{t \sin(t) \cos(t)}{t(t)} dt$$

$$\underbrace{\sin(t) \cos(t)}_{g'(t)}$$

choose $g(t) = \frac{1}{2} \sin^2(t)$.

//

$$\frac{t}{2} \sin^2(t) - \left(\frac{1}{2} \right) \sin^2(t) dt = \frac{t}{2} \sin^2(t) - \frac{1}{2} \int (1 - \cos 2t) dt$$

$$= \frac{t}{2} \sin^2(t) - \frac{1}{4} \left(t - \frac{1}{2} \sin(2t) \right) + C //$$

Q14 // Substitute $x = \tan \theta \Rightarrow \frac{dx}{d\theta} = \sec^2 \theta \Rightarrow dx = \sec^2 \theta d\theta$

$\left(-\frac{\pi}{2} < \theta < \frac{\pi}{2} \right)$

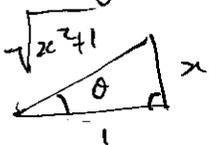
$$\Rightarrow \int \frac{x^3}{\sqrt{1+x^2}} dx = \int \frac{\tan^3 \theta}{\sec \theta} \sec^2 \theta d\theta = \int \sin^3 \theta \cos^{-4} \theta d\theta$$

(substitute $u = \cos \theta \Rightarrow \frac{du}{d\theta} = -\sin \theta \Rightarrow d\theta = \frac{1}{-\sin \theta} du$)

$$= - \int (1-u^2) u^{-4} du = \frac{-1}{-3} u^{-3} + \frac{u^{-1}}{-1} + C$$

$$= \frac{1}{3} \cos^{-3} \theta + \cos^{-1} \theta + C$$

$$= \frac{1}{3} (x^2+1)^{\frac{3}{2}} + \sqrt{x^2+1} + C //$$



$$\Rightarrow \cos \theta = \frac{1}{\sqrt{x^2+1}}$$

$$\frac{dx}{dx} = \frac{1}{2} x^{-2} \Rightarrow dx = 2u du$$

$$\Rightarrow \int \arctan(\sqrt{x}) dx = \int 2u \arctan(u) du$$

(This is just §7.4 Q54// so we get)

$$= 2 \left(\frac{1}{2} u^2 \arctan(u) - \frac{1}{2} u + \frac{1}{2} \arctan(u) \right) + C$$

$$= x \arctan(\sqrt{x}) - \sqrt{x} + \frac{2}{2} \arctan \sqrt{x} + C$$

$$= (x+1) \arctan(\sqrt{x}) - \sqrt{x} + C$$

Q24// Substitute ~~xxxx~~ $x = \sec(\theta)$ ($0 \leq \theta < \frac{\pi}{2}$ $\pi \leq \theta < \frac{3\pi}{2}$)

$$\frac{dx}{d\theta} = \tan \theta \sec \theta \Rightarrow dx = \tan \theta \sec \theta$$

$$\Rightarrow \int \ln |x + \sqrt{x^2 - 1}| dx = \int \tan \theta \sec \theta \ln |\sec \theta + \tan \theta| d\theta$$

(Now do integration by parts with $f(\theta) = \ln |\sec \theta + \tan \theta|$

$$g'(\theta) = \tan \theta \sec \theta \quad \text{Choose } g(\theta) = \sec \theta$$

$$= \sec \theta \ln |\sec \theta + \tan \theta| - \int \sec^2 \theta d\theta$$

$$= \sec \theta \ln |\sec \theta + \tan \theta| - \tan \theta + C$$

$$= x \ln |x + \sqrt{x^2 - 1}| - \sqrt{x^2 - 1} + C$$

$$\begin{aligned}
 \int_{-1}^2 |e^x - 1| dx &= \int_{-1}^0 (1 - e^x) dx + \int_0^2 (e^x - 1) dx \\
 &= \left[x - e^x \right]_{-1}^0 + \left[e^x - x \right]_0^2 \\
 &= (-1) - (-1 - e^{-1}) + ((e^2 - 2) - 1) \\
 &= e^{-1} + e^2 - 3
 \end{aligned}$$

Q47 // Substitute $u = x - 1 \Rightarrow du = dx \Rightarrow$

$$\begin{aligned}
 \int \frac{x^3}{(x-1)^4} dx &= \int \frac{(u+1)^3}{u^4} du \\
 &= \int (u^{-1} + 3u^{-2} + 3u^{-3} + u^{-4}) du \\
 &= |u|u| + (-3u^{-1}) - \frac{3}{2} u^{-2} + \frac{1}{-3} u^{-3} + C \\
 &= |u| x-1| - 3(x-1)^{-1} - \frac{3}{2} (x-1)^{-2} - \frac{1}{3} (x-1)^{-3} + C
 \end{aligned}$$

Q66 // Substitute $y = \tan(x) \Rightarrow y = \sec(x) \sin(x)$

When $x = \frac{\pi}{4}$, $y = 1$ and when $x = \frac{\pi}{3}$, $y = \sqrt{3}$

$$\Rightarrow \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\ln(\tan(x))}{\sin(x) \cos(x)} dx = \int_1^{\sqrt{3}} \frac{\ln(y)}{y} dy$$

Now substitute $u = \ln(y) \Rightarrow \frac{du}{dy} = \frac{1}{y} \Rightarrow dy = y du \Rightarrow$

$$\int_1^{\sqrt{3}} \frac{\ln(y)}{y} dy = \int_0^{\ln(\sqrt{3})} u du = \left[\frac{1}{2} u^2 \right]_0^{\ln(\sqrt{3})} = \frac{1}{8} \ln(3)^2 //$$

Q67 // Begin by multiplying and dividing by

$\sqrt{x+1} - \sqrt{x}$ to give

$$\int \frac{1}{\sqrt{x+1} + \sqrt{x}} dx = \int (\sqrt{x+1} - \sqrt{x}) dx$$
$$= \frac{2}{3} (x+1)^{\frac{3}{2}} - \frac{2}{3} x^{\frac{3}{2}} + C //$$

Q73 // $\int \frac{x + \arcsin(x)}{\sqrt{1-x^2}} dx$

$$= \int \frac{x}{\sqrt{1-x^2}} dx + \int \frac{\arcsin(x)}{\sqrt{1-x^2}} dx$$

$$\int \frac{x}{\sqrt{1-x^2}} dx = \frac{1}{2} \int \frac{1}{\sqrt{1-u}} du = -\frac{1}{2} \cdot 2 (1-u)^{\frac{1}{2}} = -\sqrt{1-x^2}$$

In the second integral substitute $v = \arcsin(x)$ to give $\left(\frac{dv}{dx} = \frac{1}{\sqrt{1-x^2}}\right)$

$$\int \frac{\arcsin(x)}{\sqrt{1-x^2}} dx = \int v dv = \frac{1}{2} v^2 + C = \frac{1}{2} \arcsin(x) + C$$

Q82// Use substitution $u = \sin(x) \Rightarrow \frac{du}{dx} = \cos(x)$

$$\Rightarrow dx = \frac{1}{\cos(x)} du$$

$$\Rightarrow \int \frac{\sin(x) \cos(x)}{\sin^4(x) + \cos^4(x)} dx = \int \frac{u du}{u^4 + (1-u^2)^2}$$

Now do the substitution $v = u^2 \Rightarrow \frac{dv}{du} = 2u \Rightarrow du = \frac{1}{2u} dv$

$$\int \frac{u du}{u^4 + (1-u^2)^2} = \frac{1}{2} \int \frac{dv}{v^2 + (1-v)^2} = \frac{1}{4} \int \frac{dv}{(v-\frac{1}{2})^2 + \frac{1}{4}}$$

Use substitution $y = v - \frac{1}{2}$ to give

$$\frac{1}{4} \int \frac{dv}{(v-\frac{1}{2})^2 + \frac{1}{4}} = \frac{1}{4} \int \frac{dy}{y^2 + \frac{1}{4}} = \frac{1}{2} \arctan(2y) + C$$

$$= \frac{1}{2} \arctan(2 \sin^2(x) - 1) + C$$

