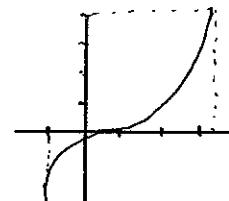


# Homework 0 Solutions

## Chapter 1 (p73, 1-3, 4-10)

- 1) a)  $f(x) \approx 2.7$  means approximately equal.  $x$  in the real numbers ( $\mathbb{R} = \text{notation for real numbers}$ )  
 b)  $x \approx 2.3$  and  $x \approx 5.6$  ✓  
 c) Domain of  $f = [-6, 6] = \{x \in \mathbb{R} | -6 \leq x \leq 6\}$   
 d) Range of  $f = [-4, 4]$   
 e)  $[-4, 4]$   
 f) NO. For example, from b) there are 2 values of  $x$  which take the value 3.  
 g)  $f$  is odd because the graph is preserved by rotation by  $\frac{\pi}{2}$  Radians  
 ( $180^\circ$ ) about  $(0,0)$ . It is not even because  $f(4) = 4$   
 $f(-4) = -4$ .

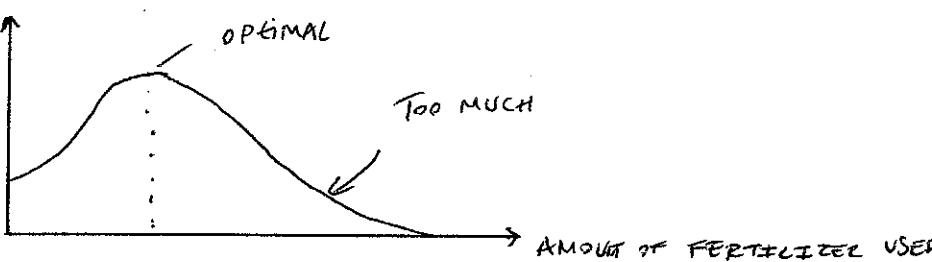
- 2) a)  $g(2) = 3$   
 b)  $g$  is one-to-one because any horizontal line crosses the graph at at most one point.  
 c)  $g^{-1}(2) \approx 0.25$   
 d) Domain of  $g^{-1} \approx [-1, 3.5]$ , e) The graph for  $g^{-1}$  is the reflection of the graph for  $g$  in the line  $y=x$ , ie.



3)  $f(x) = x^2 - 2x + 3$

$$\begin{aligned}\frac{f(a+h) - f(a)}{h} &= \frac{(a+h)^2 - 2(a+h) + 3 - a^2 + 2a - 3}{h} \\ &= \frac{a^2 + 2ah + h^2 - 2a - 2h + 3 - a^2 + 2a - 3}{h} \\ &= \frac{2ah + h^2 - 2h}{h} = 2a - 2 + h.\end{aligned}$$

4) CROP YIELD



- 5) DOMAIN =  $(-\infty, \frac{1}{3}) \cup (\frac{1}{3}, \infty)$ ; RANGE =  $(-\infty, 0) \cup (0, \infty)$   
 6) DOMAIN =  $[-2, 2]$ ; RANGE =  $[0, 4]$   
 7) DOMAIN =  $(-\infty, \infty)$ ; RANGE =  $\mathbb{R} \subseteq$  all numbers.  
 8) DOMAIN =  $\mathbb{R}$ ; RANGE =  $[2, 4]$   
 9) a) Translate graph 8 units upwards; b) Translate graph 8 units to left;  
 c) Stretch graph vertically by a factor of 2 then translate 1 unit upwards;  
 d) Translate graph 2 units to right and 2 units downwards;  
 e) Reflect graph about  $x$ -axis; f) Reflect graph about line  $y=x$ .  
 10) a) DRAW same graph shifted 8 to right; b) draw same graph reflected in  $x$ -axis; c) SAME AS b) but translate 2 upwards; d) vertically scale graph by  $\frac{1}{2}$  and translate 1 downwards; e) Reflect in  $y=x$   
 f) Reflect in  $y=x$  and translate 8 to left. YOU CAN DRAW THESE PICTURES.

## CHAPTER 2 (P167, 1, 3-6, 42, 43)

1) a) 3; b) 0; c) Does NOT EXIST; d)  $e$ ; e)  $\infty$ ; f)  $-\infty$ ; g) 4; h) -1  
 b)  $y=4, y=-1$ ; c)  $x=0, x=2$ ; d)  $-3, 0, 2, 4$   
 3) 1; 4) 0; 5)  $\frac{x^2-9}{x^2+2x-3} = \frac{(x+3)(x-3)}{(x+3)(x-1)} = \frac{x-3}{x-1} \Rightarrow \lim_{x \rightarrow -3} \frac{x^2-9}{x^2+2x-3} = \frac{-6}{-4} = \frac{3}{2}$   
 6)  $-\infty$   
 42)   
 43)

## CHAPTER 3 (P265, 4-18)

9) (CHAIN RULE + PRODUCT RULE)  $y = \ln(u)$   $u = x \ln(x)$   
 $y' = \frac{dy}{dx} = \frac{d\ln(u)}{du} \cdot \frac{du}{dx} = \frac{1}{u} \cdot \frac{d(x \ln x)}{dx} = \frac{1}{x \ln(x)} \left( \ln(x) + x \cdot \left(\frac{1}{x}\right) \right)$   
 $= \frac{\ln(x) + 1}{x \ln(x)}$   
 10)  $y' = m e^{mx} \cos(nx) + n e^{mx} (-\sin(nx))$   
 11)  $y' = \frac{1}{2} x^{-\frac{1}{2}} \cos(x^{\frac{1}{2}}) + x^{\frac{1}{2}} \cdot \left(\frac{1}{2} x^{-\frac{1}{2}}\right) (-\sin(x^{\frac{1}{2}}))$   
 $= \frac{\cos(\sqrt{x})}{2\sqrt{x}} + \frac{1}{2} \sin(\sqrt{x})$

$$12) \quad y' = 2 \arcsin(2x) + 2 \cdot \frac{1}{\sqrt{1-(2x)^2}} = \frac{4 \arcsin(2x)}{\sqrt{1-4x^2}}$$

$$13) \quad y' = \left(\frac{1}{x^2}\right) \cdot \left(\frac{-1}{x^2} e^{1/x}\right) + \frac{-2}{x^3} \cdot e^{1/x}$$

$$= \frac{-e^{1/x} - 2xe^{1/x}}{x^4}$$

$$14) \quad y' = \frac{1}{\sec(x)} \cdot (\tan(x) \sec(x)) = \tan(x)$$

15) (Implicit differentiation)

$$\frac{dy}{dx} + \frac{d(x \cos y)}{dx} = \frac{d(x^2 y)}{dx}$$

$$\Rightarrow \frac{dy}{dx} + \cos y \frac{dx}{dx} + x \frac{d \cos y}{dx} = x^2 \frac{dy}{dx} + y \frac{d x^2}{dx}$$

$$\Rightarrow \frac{dy}{dx} + \cos y + x(-\sin y) \frac{dy}{dx} = x^2 \frac{dy}{dx} + 2xy$$

$$\Rightarrow \frac{dy}{dx} (1 - x \sin(y) - x^2) = 2xy - \cos(y)$$

$$\Rightarrow \frac{dy}{dx} = \frac{2xy - \cos(y)}{1 - x \sin(y) - x^2}$$

$$16) \quad y' = \frac{dy}{du} = 4 \left( \frac{(u-1)}{u^2+u+1} \right)^3 \cdot \left( (u-1) \left( (2u+1) \left( \frac{-1}{(u^2+u+1)^2} \right) + \frac{1}{u^2+u+1} \right) \right.$$

$$= 4 \frac{(u-1)^3}{(u^2+u+1)^3} \cdot \left( \frac{(1-u)(2u+1) + (u^2+u+1)}{(u^2+u+1)^2} \right)$$

$$= 4 \frac{(u-1)^3 (-u^2+2u+2)}{(u^2+u+1)^5}$$

$$17) \quad y' = \frac{1}{2} (\arctan(u))^{\frac{1}{2}} \cdot \frac{1}{1+x^2} = \frac{1}{2\sqrt{\arctan(u)} (1+x^2)}$$

$$18) \quad y' = -\csc^2(\csc(x)) \cdot (-\cot(x) \csc(x)) .$$

CHAPTER 4 (p 352, 1-3, 9-12)

$$1) \quad f'(x) = 3x^2 - 12x + 9 = 3(x-1)(x-3) \Rightarrow$$

stationary points at  $x=1$  and  $x=3$ .  $f(2) = 3, f(1) = 1, f(4) = 5$   
 not in domain

$\Rightarrow$  Abs Max at  $x=4$ , Abs and Loc min at  $x=3$ .

2)  $f'(x) = \frac{1 - \frac{3}{2}x}{\sqrt{1-x}} \Rightarrow$  stationary point at  $x = \frac{3}{2}$

$f(-1) = -\sqrt{2}$ ,  $f(1) = 0$ ,  $f(\frac{2}{3}) = \frac{2}{3\sqrt{3}}$   $\Rightarrow$  Abs and Loc max at  $x = \frac{2}{3}$ ; Abs min at  $x = -1$

3) SAME PROCESS AS ABOVE GIVES: Abs Max at  $x=2$  ( $f(2) = \frac{2}{5}$ ) ;  
Abs and loc min at  $x = \frac{-1}{3}$  ( $f(-\frac{1}{3}) = -\frac{4}{27}$ )

4)  $e^{4x} - 1 - 4x = 1 - 1 = 0$ . Hence use L'Hopital's rule.

$$\lim_{x \rightarrow 0} \frac{e^{4x} - 1 - 4x}{x^2} = \lim_{x \rightarrow 0} \frac{4e^{4x} - 4}{2x} = \lim_{x \rightarrow 0} \frac{16e^{4x}}{2} = 8$$

*L'Hopital again*

5) Again use L'Hopital:

$$\lim_{x \rightarrow \infty} \frac{e^{4x} - 1 - 4x}{x^2} = \lim_{x \rightarrow \infty} \frac{4e^{4x} - 4}{2x} = \lim_{x \rightarrow \infty} \frac{16e^{4x}}{2} = \infty.$$

6)  $(x^2 - x^3) e^{-2x} = \frac{x^2 - x^3}{e^{-2x}}$ . Now can apply L'Hopital's rule.

$$\begin{aligned} \lim_{x \rightarrow -\infty} \frac{x^2 - x^3}{e^{-2x}} &= \lim_{x \rightarrow -\infty} \frac{2x - 3x^2}{-2e^{-2x}} = \lim_{x \rightarrow -\infty} \frac{2 - 6x}{4e^{-2x}} = \lim_{x \rightarrow -\infty} \frac{-6}{-8e^{-2x}} \\ &= 0 \quad \text{as} \quad e^{-2x} \rightarrow \infty \quad \text{as} \quad x \rightarrow -\infty. \end{aligned}$$

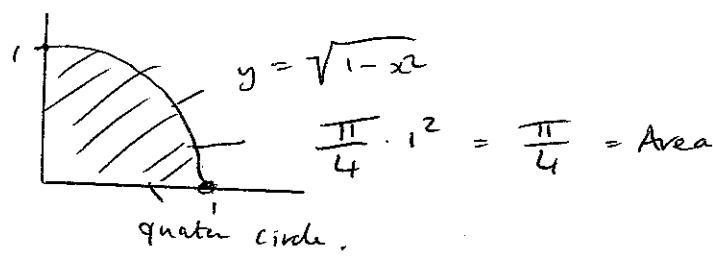
7)  $(x - \pi) \csc x = \frac{x - \pi}{\sin x}$ . Apply L'Hopital

$$\lim_{x \rightarrow \pi^-} \frac{x - \pi}{\sin x} = \lim_{x \rightarrow \pi^-} \frac{1}{\cos x} = \frac{1}{-1} = -1.$$

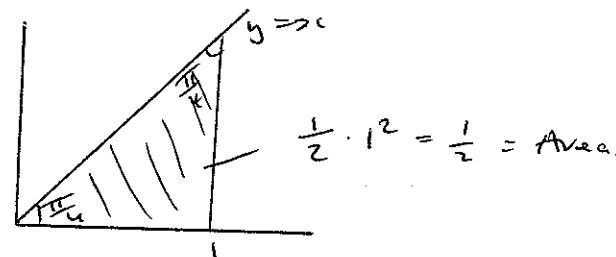
## CHAPTER 5 (P 416 / 417, 1, 3, 9-18, 27-38)

1) SEE TEXTBOOK

3) PICTURE:



$$\Rightarrow \int_0^1 x + \sqrt{1-x^2} dx = \frac{1}{2} + \frac{\pi}{4}$$



$$9) \int_1^2 (8x^3 + 3x^2) dx = \int_1^2 \left( \frac{8}{4}x^4 + \frac{3}{3}x^3 \right) dx \\ = \int_1^2 (2x^4 + x^3) dx = (2^5 + 2^3) - (2+1) = 37$$

$$10) \int_0^T (x^6 - 8x + 7) dx = \int_0^T \left( \frac{1}{5}x^5 - \frac{8}{2}x^2 + 7x \right) dx \\ = \frac{1}{5}T^5 - 4T^2 + 7T$$

$$11) \int_0^1 (1-x^9) dx = \int_0^1 \left( 1 - \frac{1}{10}x^{10} \right) dx = 1 - \frac{1}{10} = \frac{9}{10}$$

$$12) \int_0^1 (1-x)^9 dx = \int_0^1 \frac{1}{10}(1-x)^{10} dx = \frac{1}{10}$$

$$13) \int_1^9 \frac{\sqrt{u}-2u^2}{u} du = \int_1^9 \left( u^{-\frac{1}{2}} - 2u \right) du \\ = \int_1^9 (2u^{-\frac{1}{2}} - u^2) du = (6-81) - (2-1) \\ = -75 - 1 = -76$$

$$14) \int_0^1 (\sqrt[4]{u} + 1)^2 du = \int_0^1 (\sqrt[4]{u} + \sqrt[4]{u} + 1) du \\ = \int_0^1 \left( \frac{2}{3}u^{3/2} + \frac{8}{5}u^{5/4} + u \right) du = \frac{2}{3} + \frac{8}{5} + 1 \\ = \frac{10+24+15}{15} = \frac{49}{15}$$

$$15) \int_0^1 y(y^2+1)^5 dy . \text{ Let } u = y^2+1 \Rightarrow \frac{du}{dy} = 2y \Rightarrow dy = \frac{1}{2y} du$$

$$\Rightarrow \int_0^1 y(y^2+1)^5 dy = \int_{0^2+1}^{1^2+1} \frac{1}{2}u^5 du = \int_1^2 \frac{1}{12}u^6 du = \frac{1}{12}(2^6 - 1)$$

$$16) \int_0^2 y^2 \sqrt{1+y^3} dy . \text{ Let } u = 1+y^3 \Rightarrow \frac{du}{dy} = 3y^2 \Rightarrow dy = \frac{1}{3y^2} du$$

$$\Rightarrow \int_0^2 y^2 \sqrt{1+y^3} dy = \int_{1+0^2}^{1+2^2} \frac{1}{3}\sqrt{u} du = \int_1^4 \frac{2}{9}u^{3/2} du = \frac{2}{9}(27-1) \\ = \frac{52}{9}$$

(7) Observe that  $\frac{1}{(t-4)^2} \rightarrow \infty$  as  $t \rightarrow 4$  and  $4 \in [1, 5]$

Observe that the anti-derivative of  $\frac{1}{(t-4)^2}$  is  $\frac{-1}{(t-4)}$ .

If  $\int_1^s \frac{dt}{(t-4)^2}$  made sense then so would  $\int_4^s \frac{dt}{(t-4)^2}$ .

But this would be equal to  $\int_4^s \frac{-1}{(t-4)} dt$ . This does not make sense as  $\frac{-1}{t-4}$  is not defined at  $t=4$ . Hence  $\int_1^s \frac{dt}{(t-4)^2}$  does not exist.

$$(8) \int_0^1 \sin(3\pi t) dt = \left[ \frac{-1}{3\pi} \cos(3\pi t) \right]_0^1 = \frac{(-1) \cdot (-1)}{3\pi} - \left( \frac{-1}{3\pi} \cdot 1 \right) = \frac{2}{3\pi}$$

$$27) \sin \pi t \cos \pi t = \frac{1}{2} \sin 2\pi t$$

$$\Rightarrow \int \sin \pi t \cos \pi t dt = \frac{-1}{4\pi} \cos 2\pi t + C$$

$$28) \int \sin x \cos(\cos x) dx. \text{ Let } u = \cos x \quad \frac{du}{dx} = -\sin x \Rightarrow dx = \frac{-1}{\sin x} du$$

$$\Rightarrow \int \sin x \cos(\cos x) dx = \int -\cos(u) du = -\sin(u) + C = -\sin(\cos(x)) + C$$

$$29) \text{ Let } u = \sqrt{x} \Rightarrow \frac{du}{dx} = \frac{1}{2\sqrt{x}} \Rightarrow dx = 2\sqrt{x} du$$

$$\Rightarrow \int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = \int 2e^u du = 2e^u + C = 2e^{\sqrt{x}} + C$$

$$30) \text{ Let } u = \ln(x) \Rightarrow \frac{du}{dx} = \frac{1}{x} \Rightarrow dx = x du$$

$$\Rightarrow \int \frac{\cos(\ln(x))}{x} dx = \int \cos u du = \sin u + C = \sin(\ln(x)) + C$$

$$31) \text{ Let } u = \ln(\cos x) \Rightarrow \frac{du}{dx} = \frac{-\sin x}{\cos x} = -\tan x \Rightarrow dx = \frac{-1}{\tan x} du$$

$$\Rightarrow \int \tan(x) \ln(\cos(x)) dx = \int -u du = \frac{-1}{2} u^2 + C = \frac{-1}{2} (\ln(\cos(x))^2 + C)$$

32) Observe that  $1 - x^4 = (1-x^2)(1+x^2)$ . Let  $u = x^2$   
 $\Rightarrow \frac{du}{dx} = 2x \Rightarrow dx = \frac{1}{2x} du$

$$\Rightarrow \int \frac{x}{\sqrt{1-x^4}} dx = \int \frac{1}{2} \cdot \frac{1}{\sqrt{1-u^2}} du = \frac{1}{2} \arcsin(u) + C$$

$$= \frac{1}{2} \arcsin(x^2) + C.$$

33) Let  $u = x^4 \Rightarrow \frac{du}{dx} = 4x^3 \Rightarrow dx = \frac{1}{4x^3} du$

$$\Rightarrow \int \frac{x^3}{1+x^4} dx = \int \frac{1}{4} \cdot \frac{1}{1+u} du = \frac{1}{4} \ln(1+u) + C$$

$$= \frac{1}{4} \ln(1+x^4) + C$$

34)  $\int \sinh(1+4x) dx = \frac{1}{4} \cosh(1+4x) + C$

35) Let  $u = \sec \theta \Rightarrow \frac{du}{d\theta} = \tan \theta \sec \theta \Rightarrow d\theta = \frac{1}{\tan \theta \sec \theta} du$

$$\Rightarrow \int \frac{\sec \theta \tan \theta}{1+\sec \theta} d\theta = \int \frac{1}{1+u} du = \ln(1+u) + C$$

$$= \ln(1+\sec \theta) + C.$$

36) Let  $u = \tan t \Rightarrow \frac{du}{dt} = \sec^2 t \Rightarrow dt = \frac{1}{\sec^2 t} du$

$$\Rightarrow \int_0^{\frac{\pi}{4}} (1+\tan t)^3 \sec^2 t dt = \int_{\tan(0)}^{\tan(\frac{\pi}{4})} (1+u)^3 dt = \int_0^1 \frac{1}{4} (1+u)^4$$

$$= 4 - \frac{1}{4} = \frac{15}{4}$$

37)  $\int_8^3 |x^2 - 4| dx = \int_1^2 |x^2 - 4| dx + \int_2^3 |x^2 - 4| dx$

$$= \int_0^2 (4 - x^2) dx + \int_2^3 x^2 - 4 dx$$

$$= \int_0^2 \left(4x - \frac{1}{3}x^3\right) dx + \int_2^3 \left(\frac{1}{3}x^3 - 4x\right) dx$$

$$= 8 - \frac{8}{3} + \left(\left(\frac{1}{3}x^4\right)_2^3 - \left(4x^2\right)_2^3\right) = 16 - \frac{16}{3} - 3 = 13 - \frac{16}{3} = \frac{23}{3}$$

$$\begin{aligned}
 38) \quad \int_0^4 |\sqrt{x} - 1| dx &= \int_0^4 (1 - \sqrt{x}) dx + \int_1^4 (\sqrt{x} - 1) dx \\
 &= \left[ \left( x - \frac{2}{3}x^{\frac{3}{2}} \right) \right]_0^4 + \left[ \left( \frac{2}{3}x^{\frac{3}{2}} - x \right) \right]_1^4 \\
 &= \left( 1 - \frac{2}{3} \right) + \left[ \left( \frac{16}{3} - 4 \right) - \left( \frac{2}{3} - 1 \right) \right] \\
 &= \frac{1}{3} + \left[ \frac{4}{3} + \frac{1}{3} \right] \\
 &= \frac{6}{3} = 2.
 \end{aligned}$$