DO NOT TURN OVER UNTIL INSTRUCTED TO DO SO.

Formulae

$$\int \tan(x)\ dx = \ln|\sec(x)| + C \qquad \int \sec(x)\ dx = \ln|\sec(x) + \tan(x)| + C$$

$$\int \frac{1}{1+x^2}dx = \arctan(x) + C \qquad \int \frac{1}{\sqrt{1-x^2}}dx = \arcsin(x) + C$$

$$\frac{d\tan(x)}{dx} = \sec^2(x) \qquad \frac{d\sec(x)}{dx} = \tan(x)\sec(x)$$

$$1 = \sin^2(x) + \cos^2(x) \qquad 1 + \tan^2(x) = \sec^2(x)$$

$$\cos^2(x) = \frac{1+\cos(2x)}{2} \qquad \sin^2(x) = \frac{1-\cos(2x)}{2}$$

$$e^x = 1 + x + \tfrac{x^2}{2} + \tfrac{x^3}{6} + \cdots = \textstyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\sin x = x - \tfrac{x^3}{3!} + \tfrac{x^5}{5!} - \tfrac{x^7}{7!} + \tfrac{x^9}{9!} - \cdots = \textstyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$\cos x = 1 - \tfrac{x^2}{2!} + \tfrac{x^4}{4!} - \tfrac{x^6}{6!} + \tfrac{x^8}{8!} - \cdots = \textstyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

$$\arctan x = x - \tfrac{x^3}{3} + \tfrac{x^5}{5} - \tfrac{x^7}{7} + \tfrac{x^9}{9} - \cdots = \textstyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$$

CALCULATORS ARE NOT PERMITTED

This exam consists of 10 questions. Answer the questions in the spaces provided.

Name and section: _____

GSI's name: _____

1. Compute the following integrals:

   (a) (5 points)

$$\int x \ln(x+1)\, dx$$

   Solution:

   (b) (5 points)

$$\int x^3 \sqrt{x^2+4}\, dx$$

   Solution:

2. (10 points) Determine if the following series are absolutely convergent, conditionally convergent or divergent. You do not need to show your working.

(a)
$$\sum_{n=1}^{\infty}(\arctan(n+1)-\arctan(n))$$

Solution:

(b)
$$\sum_{n=1}^{\infty}\frac{n}{n^3+2}$$

Solution:

(c)
$$\sum_{n=1}^{\infty}\frac{(2n)!}{(3n)!}$$

Solution:

(d)
$$\sum_{n=1}^{\infty}\frac{1+4^n}{2^n+3^n+4^n}$$

Solution:

(e)
$$\sum_{n=1}^{\infty}(-1)^n\sin(\frac{1}{n})$$

Solution:

3. Determine the radius and interval of convergent of the following power series:

(a) (5 points)

$$\sum_{n=1}^{\infty} \frac{2^n}{n}(x-4)^{2n}$$

Solution:

(b) (5 points)

$$\sum_{n=1}^{\infty} \frac{n!}{(4n)!}x^n$$

Solution:

4. (10 points) What is the domain of the function $f(x)$ given by the power series

$$\sum_{n=1}^{\infty} \frac{(x-3)^n}{12^n n}$$

What is the value of $f^{(3)}(3)$?

Solution:

5. (10 points) Determine the Taylor series of the function

$$\frac{1}{(1-x)^2},$$

about the point $x = \frac{1}{2}$.

Solution:

6. (10 points) Find the general solution to the following differential equation

$$x\frac{dy}{dx} = y - x.$$

Solution:

7. (10 points) Find the equation of the curve which passes through (1,0) and is orthogonal to the family of curves given by $y^2 = kx^3$, for k a constant.

Solution:

8. (10 points) Find a solution to the initial-value problem

$$\sec(x)\frac{dy}{dx} - xe^{-y} = 0, \quad y(0) = 2$$

Solution:

9. (10 points) Find the general solution to the following differential equation

$$y'' + 2y' + 2y = x\sin(2x) + x^2$$

Solution:

10. (10 points) Find a non-zero power series solution to the following differential equation

$$y'' - xy = 0.$$

You do not need to show convergence.

Solution:

END OF EXAM