## DO NOT TURN OVER UNTIL INSTRUCTED TO DO SO.

In this exam you may assume, without justification the following identity:

$$\lim_{n \to \infty} (1 + \frac{1}{n})^n = e$$



This exam consists of 5 questions. Answer the questions in the spaces provided.

Name and section: \_\_\_\_\_\_\_GSI's name: \_\_\_\_\_\_

- 1. Determine if the following sequences converge or diverge. Carefully justify your answer.
  - (a) (10 points)

 $\Big\{\frac{\cos(n)}{\sqrt{n}}\Big\}_{n=1}^{\infty}$ 

Solution:

(b) (10 points)

$$\left\{n\sin(\frac{1}{n})\right\}_{n=1}^{\infty}$$

- 2. Determine whether the following series are convergent or divergent. If convergent determine the sum.
  - (a) (10 points)

$$\sum_{n=1}^{\infty} \ln(\frac{n}{n+1})$$

(Hint: Try to explicitly determine the partial sums) Solution:

(b) (10 points)

$$\sum_{n=1}^{\infty} \frac{\sqrt{4n^2 + 2n + 1}}{4n + 6}$$

3. (20 points) Determine whether the following series is convergent or divergent. If convergent you do not need to determine the sum.

$$\sum_{n=1}^{\infty} \frac{\sin(n)}{7^n - 3^n}$$

4. (20 points) Determine whether the following series is absolutely convergent, conditionally convergent or divergent. If convergent you do not need to determine the sum.

$$\sum_{n=1}^{\infty} (-1)^{n-1} n e^{-n^2} = e^{-1} - 2e^{-4} + 3e^{-9} + \cdots$$

5. (20 points) Determine whether the following series is convergent or divergent. If convergent you do not need to determine the sum.

$$\sum_{n=1}^{\infty} \frac{n^n}{(2n)!}$$