DO NOT TURN OVER UNTIL INSTRUCTED TO DO SO.

In this exam you may assume, without justification the following identity:

$$\lim_{n\to\infty}(1+\frac{1}{n})^n = e$$

CALCULATORS ARE NOT PERMITTED

YOU MAY USE YOUR OWN BLANK PAPER FOR ROUGH WORK

SO AS NOT TO DISTURB OTHER STUDENTS, EVERYONE MUST STAY UNTIL THE EXAM IS COMPLETE

REMEMBER THIS EXAM IS GRADED BY A HUMAN BEING. WRITE YOUR SOLUTIONS NEATLY AND COHERENTLY, OR THEY RISK NOT RECEIVING FULL CREDIT

This exam consists of 5 questions. Answer the questions in the spaces provided.

Name and section: ____________________

GSI's name: ____________________

1. Determine if the following sequences converge or diverge. Carefully justify your answer.

   (a) (10 points)

$$\left\{\frac{e^{-n}}{\sin(\frac{1}{n})}\right\}_{n=1}^{\infty}$$

   Solution:

   (b) (10 points)

$$\left\{\frac{1}{2+(-1)^n}\right\}_{n=1}^{\infty}$$

   Solution:

2. (20 points) Using the integral test, prove the following series is convergent

$$\sum_{n=1}^{\infty} \frac{e^{\frac{1}{n}}}{n^2}.$$

Using this, prove that

$$\sum_{n=1}^{\infty} \frac{e^{\frac{1}{n}}}{n^3+1}$$

is convergent.

Solution:

3. (20 points) Determine if the following series is convergent or divergent

$$\sum_{n=1}^{\infty} \frac{(n-1)2^{\sin(n^2)}}{n^4+3n+1}$$

Solution:

4. Determine whether the following series are convergent or divergent. If convergent determine the sum.

   (a) (10 points)

$$\sum_{n=1}^{\infty} n \tan\left(\frac{1}{n}\right)$$

   Solution:

   (b) (10 points)

$$\sum_{n=1}^{\infty} \frac{10^n + 5^n}{6^n + 4^n + 3^n}$$

   Solution:

5. (20 points) Determine whether the following series is absolutely convergent, conditionally convergent or divergent.

$$\sum_{n=1}^{\infty}(-1)^n \frac{n^5}{\sin(\frac{1}{n})n!}$$

Solution:

END OF EXAM