

**DO NOT TURN OVER UNTIL  
INSTRUCTED TO DO SO.**

**Formulae**

$$\begin{array}{ll} \int \tan(x) \, dx &= \ln |\sec(x)| + C & \int \sec(x) \, dx &= \ln |\sec(x) + \tan(x)| + C \\ \int \frac{1}{1+x^2} dx &= \arctan(x) + C & \int \frac{1}{\sqrt{1-x^2}} dx &= \arcsin(x) + C \\ \frac{d \tan(x)}{dx} &= \sec^2(x) & \frac{d \sec(x)}{dx} &= \tan(x) \sec(x) \\ 1 &= \sin^2(x) + \cos^2(x) & 1 + \tan^2(x) &= \sec^2(x) \\ \cos^2(x) &= \frac{1 + \cos(2x)}{2} & \sin^2(x) &= \frac{1 - \cos(2x)}{2} \\ |E_T| &\leq \frac{K(b-a)^3}{12n^2} & |E_S| &\leq \frac{K(b-a)^5}{180n^4} \end{array}$$

**CALCULATORS ARE NOT PERMITTED**

**This exam consists of 5 questions. Answer the questions in the spaces provided.**

Name and section: \_\_\_\_\_

GSI's name: \_\_\_\_\_

1. Compute the following integrals:

(a) (10 points)

$$\int \ln(x)^2 \, dx$$

**Solution:**

(b) (10 points)

$$\int \tan^5(x) \sec^{-3}(x) \, dx$$

**Solution:**

2. (20 points) Find the arc length of the the curve  $y = \ln(\cos(x))$  between 0 and  $\frac{\pi}{3}$ .

**Solution:**

3. (20 points) Compute the following integral:

$$\int \frac{x^3 + x^2 - x + 1}{(x-1)^2(x^2+1)} dx$$

**Solution:**

4. (a) (10 points) Use the Trapezoidal Rule with  $n=4$  to approximate the definite integral

$$\int_0^8 f(x) \, dx,$$

where  $f(x)$  takes the following values:

$x$	0	1	2	3	4	5	6	7	8
$f(x)$	0	2	4	3	1	4	5	5	3

**Solution:**

- (b) (10 points) Assuming that  $|f''(x)| \leq 2$ , for all  $0 < x < 8$ , how large an  $n$  would we need to choose to guarantee that

$$|E_T| \leq 0.01$$

**Solution:**

5. (20 points) Evaluate following improper integral:

$$\int_{-1}^0 \frac{(x+1)^5}{\sqrt{(-x^2-2x)}} dx$$

If it is divergent, write divergent and explain your reasoning.

**Solution:**