DO NOT TURN OVER UNTIL INSTRUCTED TO DO SO.

Formulae

$$\int \tan(x)\ dx = \ln|\sec(x)| + C \qquad \int \sec(x)\ dx = \ln|\sec(x)+\tan(x)| + C$$

$$\int \frac{1}{1+x^2}dx = \arctan(x) + C \qquad \int \frac{1}{\sqrt{1-x^2}}dx = \arcsin(x) + C$$

$$\frac{d\tan(x)}{dx} = \sec^2(x) \qquad \frac{d\sec(x)}{dx} = \tan(x)\sec(x)$$

$$1 = \sin^2(x) + \cos^2(x) \qquad 1 + \tan^2(x) = \sec^2(x)$$

$$\cos^2(x) = \frac{1+\cos(2x)}{2} \qquad \sin^2(x) = \frac{1-\cos(2x)}{2}$$

$$e^x = 1 + x + \tfrac{x^2}{2} + \tfrac{x^3}{6} + \cdots = \textstyle\sum_{n=0}^{\infty} \tfrac{x^n}{n!}$$

$$\sin x = x - \tfrac{x^3}{3!} + \tfrac{x^5}{5!} - \tfrac{x^7}{7!} + \tfrac{x^9}{9!} - \cdots = \textstyle\sum_{n=0}^{\infty}(-1)^n \tfrac{x^{2n+1}}{(2n+1)!}$$

$$\cos x = 1 - \tfrac{x^2}{2!} + \tfrac{x^4}{4!} - \tfrac{x^6}{6!} + \tfrac{x^8}{8!} - \cdots = \textstyle\sum_{n=0}^{\infty}(-1)^n \tfrac{x^{2n}}{(2n)!}$$

$$\arctan x = x - \tfrac{x^3}{3} + \tfrac{x^5}{5} - \tfrac{x^7}{7} + \tfrac{x^9}{9} - \cdots = \textstyle\sum_{n=0}^{\infty}(-1)^n \tfrac{x^{2n+1}}{2n+1}$$

$$\ln(1+x) = x - \tfrac{x^2}{2} + \tfrac{x^3}{3} - \tfrac{x^4}{4} + \tfrac{x^5}{5} - \cdots = \textstyle\sum_{n=1}^{\infty}(-1)^{n-1} \tfrac{x^n}{n}$$

$$\lim_{n\to\infty}\left(\frac{n+1}{n}\right)^n = e$$

CALCULATORS ARE NOT PERMITTED

This exam consists of 10 questions. Answer the questions in the spaces provided.

Name and section: ____________________

GSI's name: ____________________

1. Compute the following integrals:

   (a) (5 points)

$$\int \sin(\sqrt{x})dx$$

   Hint: Try a substitution first.

   Solution:

(b) (5 points)

$$\int x^3\sqrt{x^2-4}\,dx$$

Solution:

2. (10 points) Determine if the following series are absolutely convergent, conditionally convergent or divergent. You do not need to show your working.

(a)

$$\sum_{n=1}^{\infty}(-1)^n \sec(\frac{1}{n^3})$$

Solution:

(b)

$$\sum_{n=1}^{\infty} e^{-n}$$

Solution:

(c)

$$\sum_{n=1}^{\infty} \frac{n!}{(3n)!}$$

Solution:

(d)

$$\sum_{n=1}^{\infty}(-1)^n \frac{\sqrt{n}}{n+1}$$

Solution:

(e)

$$\sum_{n=1}^{\infty} \frac{n^{3n}}{(2+3n^3)^n}$$

Solution:

3. Determine the radii of convergence of the following power series:

   (a) (5 points)

$$\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2}(x-2)^{2n}$$

   Solution:

   (b) (5 points)

$$\sum_{n=1}^{\infty} \frac{9^n}{n^3}x^n$$

   Solution:

4. (10 points) What is the domain of the function $f(x)$ given by the power series

$$\sum_{n=1}^{\infty} \frac{n(x-1)^n}{3^n}$$

What is the value of $f^{(5)}(1)$?

Solution:

5. (10 points) Determine the Taylor series of the function

$$f(x) = \frac{1}{\sqrt{2x-1}},$$

about the point $x = 1$.

Solution:

6. (10 points) Find a solution to the initial-value problem

$$\csc^2(y)\frac{dy}{dx} + x^2\sec^3(y) = 0, \quad y(1) = \pi/3$$

Solution:

7. (10 points) Solve the initial-value problem

$$x^2\frac{dy}{dx} + 3xy = \frac{\cos(x)}{x}, \quad y(\pi/4) = 0$$

Solution:

8. (10 points) Find the general equation for a curve which is orthogonal to the family of curves given by $(x-1)^2 = k^2 - 2y^2$, for k a constant.

   Solution:

9. (10 points) Find the general solution to the following differential equation

$$y'' - 4y' + 4y = xe^{2x}$$

Solution:

10. (10 points) Using power series methods find a non-zero solution to the following differential equation

$$x^2y'' - 2y = 0.$$

Solution:

END OF EXAM