MATH 54 MIDTERM 2 (PRACTICE 3)

PROFESSOR PAULIN

DO NOT TURN OVER UNTIL INSTRUCTED TO DO SO.

CALCULATORS ARE NOT PERMITTED

YOU MAY USE YOUR OWN BLANK PAPER FOR ROUGH WORK

SO AS NOT TO DISTURB OTHER STUDENTS, EVERYONE MUST STAY UNTIL THE EXAM IS COMPLETE

REMEMBER THIS EXAM IS GRADED BY A HUMAN BEING. WRITE YOUR SOLUTIONS NEATLY AND COHERENTLY, OR THEY RISK NOT RECEIVING FULL CREDIT

THIS EXAM WILL BE ELECTRONICALLY SCANNED. MAKE SURE YOU WRITE ALL SOLUTIONS IN THE SPACES PROVIDED. YOU MAY WRITE SOLUTIONS ON THE BLANK PAGE AT THE BACK BUT BE SURE TO CLEARLY LABEL THEM

Name and Student ID: ______________________________

GSI's name: ______________________________

This exam consists of 5 questions. Answer the questions in the spaces provided.

1. (25 points) (a) Find a basis for the subspace of all vectors $\begin{pmatrix} a-2b+5c \\ 2a+5b-8c \\ -a-4b+7c \\ 3a+b+c \end{pmatrix}$ in $\mathbb{R}^4$.

Solution:

(b) What dimension is this subspace?

Solution:

2. (a) Let $A = (\underline{\mathbf{a}}_1, \underline{\mathbf{a}}_2, \underline{\mathbf{a}}_3)$ be a 4×3 matrix with $A^T A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & -1 \\ 3 & -1 & 9 \end{pmatrix}$. Calculate $||\underline{\mathbf{a}}_1 + \underline{\mathbf{a}}_2 + \underline{\mathbf{a}}_3||$ and

Solution:

(b) Does there exist a 4×3 matrix B such that $B^T B = \begin{pmatrix} 2 & -1 & 1 \\ -1 & -1 & 3 \\ 1 & -1 & 9 \end{pmatrix}$. Justify your answer.

3. (25 points) Let V be a vector space with basis $B = \{\underline{\mathbf{b}}_1, \underline{\mathbf{b}}_2, \underline{\mathbf{b}}_3\}$. Let T be the linear transformation from V to V such that

$$T(\underline{\mathbf{b}}_1) = 4\underline{\mathbf{b}}_1 + \underline{\mathbf{b}}_3,\ T(\underline{\mathbf{b}}_2) = 4\underline{\mathbf{b}}_2 + \underline{\mathbf{b}}_3\ T(\underline{\mathbf{b}}_3) = -2\underline{\mathbf{b}}_3.$$

Find all possible λ such that there exists non-zero $\underline{\mathbf{v}}$ in V such that $T(\underline{\mathbf{v}}) = \lambda\underline{\mathbf{v}}$.

Solution:

4. Let $A = \begin{pmatrix} 7 & 4 \\ -3 & -1 \end{pmatrix}$. Given an explicit description of A^k for any positive integer k.

Solution:

5. (25 points) (a) Let $\mathbb{P}_2(\mathbb{R})$ be the vector space of polynomials of degree at most 2 with real coefficients. Let $B = \{1 + x, 1, x^2 - 1\}$ and $C = \{1 + x + x^2, 2x, 1\}$. Calculate the matrix $P_{C \leftarrow B}$.

   Solution:

   (b) Does there exist a non-zero polynomial $p(x)$ such that $(p(x))_B = (p(x))_C$?

   Solution:

END OF EXAM