MATH 54 MIDTERM 2 (PRACTICE 2)

PROFESSOR PAULIN

DO NOT TURN OVER UNTIL INSTRUCTED TO DO SO.

CALCULATORS ARE NOT PERMITTED

YOU MAY USE YOUR OWN BLANK PAPER FOR ROUGH WORK

SO AS NOT TO DISTURB OTHER STUDENTS, EVERYONE MUST STAY UNTIL THE EXAM IS COMPLETE

REMEMBER THIS EXAM IS GRADED BY A HUMAN BEING. WRITE YOUR SOLUTIONS NEATLY AND COHERENTLY, OR THEY RISK NOT RECEIVING FULL CREDIT

THIS EXAM WILL BE ELECTRONICALLY SCANNED. MAKE SURE YOU WRITE ALL SOLUTIONS IN THE SPACES PROVIDED. YOU MAY WRITE SOLUTIONS ON THE BLANK PAGE AT THE BACK BUT BE SURE TO CLEARLY LABEL THEM

Name and Student ID: ____________________

GSI's name: ____________________

This exam consists of 5 questions. Answer the questions in the spaces provided.

1. (25 points) (a) Let $\mathbb{P}_3(\mathbb{R})$ be the vector space of polynomials of degree at most 3 with real coefficients. Calculate the dimension of the subspace

$$U = \text{Span}(1 + x - x^2, 2 + x^2 + x^3, 5 - 2x - x^3, 4 - 3x + x^2 - x^3\}$$

Solution:

(b) Is $U = \mathbb{P}_3(\mathbb{R})$? Justify your answer.
Solution:

2. (25 points) (a) You are given a linear system with 5 equations in 6 unknowns. If the corresponding homogeneous linear system has a solution set spanned by two linearly independent vectors, is it true that the original linear system is guaranteed to have a solution? If it is not possible, give an explicit example of such a system.

   Solution:

   (b) What about if we instead assume that the corresponding homogeneous linear system solution set is spanned by one non-zero vector? Justify your answer.

   Solution:

3. (25 points) Let $A = \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$ and $B = \{ \begin{pmatrix} 0 \\ -1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \}$. Find a basis C such that

$$A_{B,C} = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 2 & 0 & 0 & 0 \end{pmatrix}.$$

Solution:

4. (a) Give a precise statement of what it means for a square matrix A to be diagonalizable.

Solution:

(b) Is the matrix $A = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 2 \end{pmatrix}$ diagonalizable? Justify your answer.

Solution:

5. Let W be the span of the vectors $\begin{pmatrix}1\\0\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\\-1\end{pmatrix}$ in $\mathbb{R}^4$. Find an orthogonal basis for $W^{\perp}$.

   What is the minimum distance between $\begin{pmatrix}1\\1\\1\\0\end{pmatrix}$ and W?

   Solution:

END OF EXAM