

MATH 54 MIDTERM 2 (PRACTICE 1)
PROFESSOR PAULIN

**DO NOT TURN OVER UNTIL
INSTRUCTED TO DO SO.**

CALCULATORS ARE NOT PERMITTED

**YOU MAY USE YOUR OWN BLANK
PAPER FOR ROUGH WORK**

**SO AS NOT TO DISTURB OTHER
STUDENTS, EVERYONE MUST STAY
UNTIL THE EXAM IS COMPLETE**

**REMEMBER THIS EXAM IS GRADED BY
A HUMAN BEING. WRITE YOUR
SOLUTIONS NEATLY AND
COHERENTLY, OR THEY RISK NOT
RECEIVING FULL CREDIT**

**THIS EXAM WILL BE ELECTRONICALLY
SCANNED. MAKE SURE YOU WRITE ALL
SOLUTIONS IN THE SPACES PROVIDED.
YOU MAY WRITE SOLUTIONS ON THE
BLANK PAGE AT THE BACK BUT BE
SURE TO CLEARLY LABEL THEM**

Name and Student ID: _____

GSI's name: _____

This exam consists of 5 questions. Answer the questions in the spaces provided.

1. (25 points) (a) Let V be a vector space and $U \subset V$ be a subset. Give a precise definition of what it means for U to be a subspace.

Solution:

- (b) If $V = C[0, 1]$ is the vector space of continuous real-valued functions on $[0, 1]$ is it true that

$$U = \{f(x) \text{ in } C[0, 1], \text{ such that } f(0) \text{ and } f(1) \text{ are integers}\}$$

is a subspace? Justify your answer.

Solution:

2. (25 points) Find a basis for the kernel of the following linear transformation:

$$T : \mathbb{P}_3(\mathbb{R}) \rightarrow \mathbb{R}^3$$
$$p(x) \mapsto \begin{pmatrix} p(1) \\ p'(1) \\ p''(1) \end{pmatrix}$$

Determine the rank of T . Be sure to carefully justify your answers.

Solution:

3. Let V and W be vector spaces with bases $B = \{\underline{\mathbf{b}}_1, \underline{\mathbf{b}}_2, \underline{\mathbf{b}}_3\}$ and $C = \{\underline{\mathbf{c}}_1, \underline{\mathbf{c}}_2, \underline{\mathbf{c}}_3, \underline{\mathbf{c}}_4\}$ respectively. Let T be the linear transformation from V to W such that

$$T(\underline{\mathbf{b}}_1) = \underline{\mathbf{c}}_1 - 3\underline{\mathbf{c}}_4, \quad T(\underline{\mathbf{b}}_2) = -\underline{\mathbf{c}}_3 + 2\underline{\mathbf{c}}_2, \quad T(\underline{\mathbf{b}}_3) = \underline{\mathbf{c}}_1 + \underline{\mathbf{c}}_2 + \underline{\mathbf{c}}_3.$$

Determine the nullity of T . Is T one-to-one?

Solution:

4. Let $A = \begin{pmatrix} 5 & -3 & 0 & 9 \\ 0 & 3 & 1 & -2 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$. Is it possible to find an invertible matrix P such that $P^{-1}AP$ is diagonal. If so, find such a P . If not, justify why.

Solution:

5. Let W be the span of the vectors $\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}$ in \mathbb{R}^4 . Calculate $Proj_W \begin{pmatrix} 1 \\ -1 \\ 2 \\ 1 \end{pmatrix}$ and

$$Proj_{W^\perp} \begin{pmatrix} 1 \\ -1 \\ 2 \\ 1 \end{pmatrix}?$$

Solution: