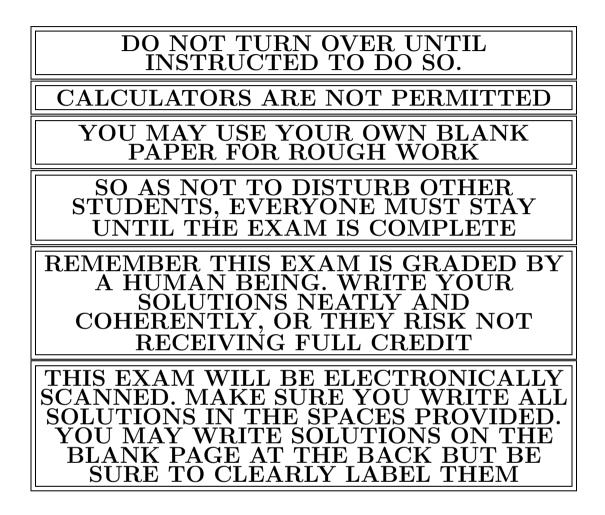
MATH 54 MIDTERM 1 (PRACTICE 3) PROFESSOR PAULIN



Name and section:

GSI's name: _____

Math 54

This exam consists of 5 questions. Answer the questions in the spaces provided.

1. (25 points) (a) Express the vector $\begin{pmatrix} -2\\0\\1 \end{pmatrix}$ as a linear combination of the vectors $\begin{pmatrix} 0\\1\\1 \end{pmatrix}, \begin{pmatrix} 2\\3\\4 \end{pmatrix}, \begin{pmatrix} 2\\1\\2 \end{pmatrix}, \begin{pmatrix} 0\\1\\2 \end{pmatrix}, \begin{pmatrix} 0\\1\\2 \end{pmatrix}.$

Solution:

$$\begin{pmatrix} 0 & 2 & 2 & 0 & | & -2 \\ 1 & 3 & 1 & 1 & | & 0 \\ 1 & 4 & 2 & 2 & | & 1 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 3 & 1 & 1 & | & 0 \\ 0 & 2 & 2 & 0 & | & -2 \\ 1 & 4 & 2 & 2 & | & 1 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 3 & 1 & 1 & | & 0 \\ 0 & 2 & 2 & 0 & | & -2 \\ 0 & 1 & 1 & | & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 3 & 1 & 0 & | & -2 \\ 0 & 1 & 1 & 0 & | & -1 \\ 0 & 0 & 0 & 1 & | & z \end{pmatrix} \leftarrow \begin{pmatrix} 1 & 3 & 1 & 1 & | & 0 \\ 0 & 1 & 1 & 0 & | & -1 \\ 0 & 0 & 0 & 1 & | & z \end{pmatrix} \leftarrow \begin{pmatrix} 1 & 3 & 1 & 1 & | & 0 \\ 0 & 1 & 1 & 0 & | & -1 \\ 0 & 1 & 1 & | & 1 \end{pmatrix}$$

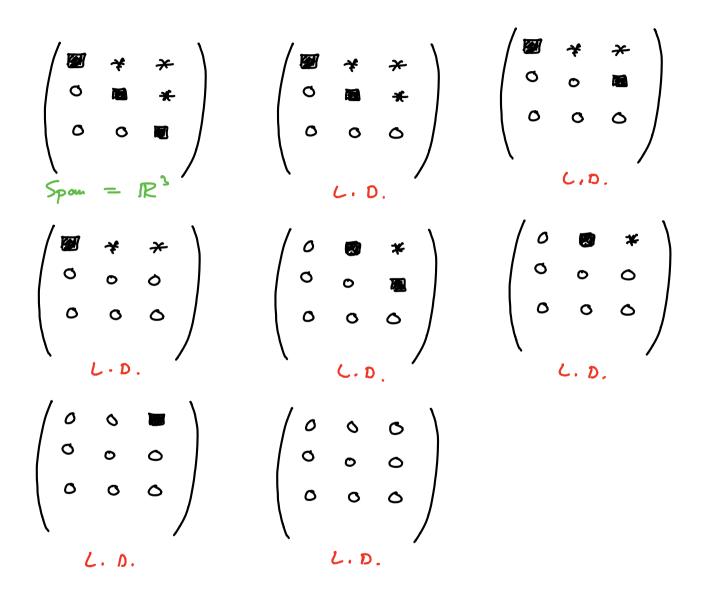
(b) How many possible ways of doing this are there? Justify your answer. Solution:

There are intinitely many ways to do this as x3 is a Tree variable.

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2. (25 points) List the possible forms of all 3×3 reduced echelon matrices. Label the ones row equivalent to matrices whose column vectors are linearly dependent. Label the ones row equivalent to matrices whose column vectors span \mathbb{R}^3 .

Solution:



3. Calculate the determinant of
$$\begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & -1 & 2 & 0 \\ 2 & 1 & 3 & 4 \\ 3 & 1 & 2 & 5 \end{pmatrix}$$

Solution:

$$\begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & -1 & 2 & 0 \\ 2 & 1 & 3 & 4 \\ 3 & 1 & 2 & 5 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & -2 & 2 & -1 \\ 0 & -1 & 3 & 2 \\ 0 & -2 & 2 & 2 \end{pmatrix} \xrightarrow{\text{Switch}} \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & -1 & 3 & 2 \\ 0 & -2 & 2 & 2 \end{pmatrix}$$

 $=) \begin{vmatrix} 1 & 1 & 0 & 1 \\ 1 & -1 & 2 & 0 \\ 2 & 0 & 3 & 4 \\ 3 & 1 & 2 & 5 \end{vmatrix} = (-1) \cdot 1 \cdot -1 \cdot -4 \cdot 3 = -12$

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4. (25 points) (a) Is it possible for a linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^3$ to be one-to one? Justify your answer. Solution:

(b) Give an example of a linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^3$ which is onto. Solution:

 $A = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$ $T_{n} reduced Form udta pirot$ $position in every now \Rightarrow T_{A} outo.$ T = TA where

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5. Let $T : \mathbb{R}^4 \to \mathbb{R}^4$ be the linear transformation given by

$$T\begin{pmatrix} x_1\\ x_2\\ x_3\\ x_4 \end{pmatrix} = \begin{pmatrix} 2x_1 + 3x_3 + x_4\\ -x_1 + x_2\\ x_3 + x_4\\ x_1 - x_4 \end{pmatrix}$$

Give an example of $\underline{\mathbf{b}}$ in \mathbb{R}^4 not in the range of T? Justify your answer. Solution:

$$T(\underline{x}) = \begin{pmatrix} 2 & 0 & 3 & | \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & -1 \end{pmatrix} \underbrace{x}$$

b not in rage of T (=> $(A | \underline{b})$ in consistent
$$\begin{pmatrix} 2 & 0 & 3 & | & b_1 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & | b_1 - 2b_4 - 3b_3 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & 0 & -1 & | & b_{11} \\ -1 & 1 & 0 & 0 & | & b_2 \\ 0 & 0 & 1 & 1 & | & b_2 \\ 2 & 0 & 3 & 1 & | & b_1 \\ 2 & 0 & 3 & 1 & | & b_1 \\ 0 & 1 & 0 & -1 & | & b_2 + b_1 \\ 0 & 0 & 1 & 1 & | & b_2 + b_1 \\ 0 & 0 & 1 & 1 & | & b_2 + b_1 \\ 0 & 0 & 0 & | & b_1 - 2b_4 - 3b_3 + 0 \\ \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & -1 & | & b_{11} \\ 0 & 1 & 0 & -1 & | & b_2 + b_1 \\ 0 & 0 & 1 & 1 & | & b_2 + b_1 \\ 0 & 0 & 3 & 3 & | & b_1 - 2b_1 - 3b_3 + 0 \\ \end{pmatrix}$$

$$Let \quad b_1 = 1, \quad b_2 = 1, \quad b_3 = 0, \quad b_4 = 0 \implies b_1 - 2b_4 - 3b_3 + 0$$

$$\Rightarrow \quad (A | \underline{b}) \quad in consistent \implies \begin{pmatrix} 1 & 0 \\ b_1 - 2b_4 - 3b_3 + 0 \\ 0 \end{pmatrix} \quad not \quad in \quad vare oth T$$