

MATH 54 MIDTERM 1 (PRACTICE 3)
PROFESSOR PAULIN

**DO NOT TURN OVER UNTIL
INSTRUCTED TO DO SO.**

CALCULATORS ARE NOT PERMITTED

**YOU MAY USE YOUR OWN BLANK
PAPER FOR ROUGH WORK**

**SO AS NOT TO DISTURB OTHER
STUDENTS, EVERYONE MUST STAY
UNTIL THE EXAM IS COMPLETE**

**REMEMBER THIS EXAM IS GRADED BY
A HUMAN BEING. WRITE YOUR
SOLUTIONS NEATLY AND
COHERENTLY, OR THEY RISK NOT
RECEIVING FULL CREDIT**

**THIS EXAM WILL BE ELECTRONICALLY
SCANNED. MAKE SURE YOU WRITE ALL
SOLUTIONS IN THE SPACES PROVIDED.
YOU MAY WRITE SOLUTIONS ON THE
BLANK PAGE AT THE BACK BUT BE
SURE TO CLEARLY LABEL THEM**

Name and section: _____

GSI's name: _____

This exam consists of 5 questions. Answer the questions in the spaces provided.

1. (25 points) (a) Express the vector $\begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}$ as a linear combination of the vectors

$$\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}.$$

Solution:

- (b) How many possible ways of doing this are there? Justify your answer.

Solution:

2. (25 points) List the possible forms of all 3×3 reduced echelon matrices. Label the ones row equivalent to matrices whose column vectors are linearly dependent. Label the ones row equivalent to matrices whose column vectors span \mathbb{R}^3 .

Solution:

3. Calculate the determinant of $\begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & -1 & 2 & 0 \\ 2 & 1 & 3 & 4 \\ 3 & 1 & 2 & 5 \end{pmatrix}$.

Solution:

4. (25 points) (a) Is it possible for a linear transformation $T : \mathbb{R}^4 \rightarrow \mathbb{R}^3$ to be one-to one? Justify your answer.

Solution:

- (b) Give an example of a linear transformation $T : \mathbb{R}^4 \rightarrow \mathbb{R}^3$ which is onto.

Solution:

5. Let $T : \mathbb{R}^4 \rightarrow \mathbb{R}^4$ be the linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 2x_1 + 3x_3 + x_4 \\ -x_1 + x_2 \\ x_3 + x_4 \\ x_1 - x_4 \end{pmatrix}.$$

Give an example of \mathbf{b} in \mathbb{R}^4 not in the range of T ? Justify your answer.

Solution: