

MATH 54 MIDTERM 1 (PRACTICE 2)
PROFESSOR PAULIN

**DO NOT TURN OVER UNTIL
INSTRUCTED TO DO SO.**

CALCULATORS ARE NOT PERMITTED

**YOU MAY USE YOUR OWN BLANK
PAPER FOR ROUGH WORK**

**SO AS NOT TO DISTURB OTHER
STUDENTS, EVERYONE MUST STAY
UNTIL THE EXAM IS COMPLETE**

**REMEMBER THIS EXAM IS GRADED BY
A HUMAN BEING. WRITE YOUR
SOLUTIONS NEATLY AND
COHERENTLY, OR THEY RISK NOT
RECEIVING FULL CREDIT**

**THIS EXAM WILL BE ELECTRONICALLY
SCANNED. MAKE SURE YOU WRITE ALL
SOLUTIONS IN THE SPACES PROVIDED.
YOU MAY WRITE SOLUTIONS ON THE
BLANK PAGE AT THE BACK BUT BE
SURE TO CLEARLY LABEL THEM**

Name and section: _____

GSI's name: _____

This exam consists of 5 questions. Answer the questions in the spaces provided.

1. (25 points) (a) Determine the value of k such $\begin{pmatrix} 2 \\ 3 \\ k \end{pmatrix}$ can be written as a linear combination of the vectors $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 2 \\ 5 \\ 3 \end{pmatrix}$. For this value express it as a linear combination.

Solution:

- (b) Give an example of a vector $\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$, such that $\left(\begin{array}{ccc|c} 1 & 0 & 2 & b_1 \\ 1 & 1 & 5 & b_2 \\ 0 & 1 & 3 & b_3 \end{array} \right)$ is inconsistent.

Solution:

2. (25 points) Show that

$$\text{Span}\left(\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ 7 \\ 14 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ 7 \end{pmatrix}\right) = \mathbb{R}^3.$$

Are these vectors linearly independent? Justify your answers.

Solution:

3. Let

$$A = \begin{pmatrix} 1 & 0 \\ -1 & 1 \\ 2 & 3 \end{pmatrix}, B = \begin{pmatrix} 1 & 2 & 1 \\ 1 & -1 & 0 \end{pmatrix}.$$

Calculate $\det(AB)$ and $\det(BA)$.

Solution:

4. A linear transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^4$ has the property that

$$T \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}, \quad T \begin{pmatrix} 0 \\ -3 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ 0 \\ 6 \end{pmatrix}, \quad T \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \\ -2 \end{pmatrix}.$$

Calculate the standard matrix associated to T . Is T one-to-one? Justify your answer.

Solution:

5. Let $T : \mathbb{R}^4 \rightarrow \mathbb{R}^4$ be the linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} x_1 + x_3 - x_4 \\ x_2 + 2x_4 \\ x_1 + 2x_4 \\ x_1 + x_3 - 2x_4 \end{pmatrix}.$$

Find a linear transformation $S : \mathbb{R}^4 \rightarrow \mathbb{R}^4$ such that $T \circ S = S \circ T = Id_{\mathbb{R}^4}$.

Solution: