

**MATH 54 MIDTERM 1 (PRACTICE 1)**  
**PROFESSOR PAULIN**

**DO NOT TURN OVER UNTIL  
INSTRUCTED TO DO SO.**

**CALCULATORS ARE NOT PERMITTED**

**YOU MAY USE YOUR OWN BLANK  
PAPER FOR ROUGH WORK**

**SO AS NOT TO DISTURB OTHER  
STUDENTS, EVERYONE MUST STAY  
UNTIL THE EXAM IS COMPLETE**

**REMEMBER THIS EXAM IS GRADED BY  
A HUMAN BEING. WRITE YOUR  
SOLUTIONS NEATLY AND  
COHERENTLY, OR THEY RISK NOT  
RECEIVING FULL CREDIT**

**THIS EXAM WILL BE ELECTRONICALLY  
SCANNED. MAKE SURE YOU WRITE ALL  
SOLUTIONS IN THE SPACES PROVIDED.  
YOU MAY WRITE SOLUTIONS ON THE  
BLANK PAGE AT THE BACK BUT BE  
SURE TO CLEARLY LABEL THEM**

Name and section: \_\_\_\_\_

GSI's name: \_\_\_\_\_

**This exam consists of 5 questions. Answer the questions in the spaces provided.**

1. (25 points) (a) Calculate the general solution to the linear system with the following augmented matrix:

$$\left( \begin{array}{ccc|c} 1 & 4 & 0 & 3 \\ 3 & 12 & 3 & 15 \\ 2 & 8 & 1 & 8 \end{array} \right)$$

**Solution:**

- (b) Write down a general solution to the associated homogeneous linear system.

**Solution:**

2. (25 points) Determine all values of  $h$  such that the vectors  $\begin{pmatrix} 1 \\ 2 \\ 0 \\ 2 \end{pmatrix}$ ,  $\begin{pmatrix} 0 \\ h+1 \\ 0 \\ h+1 \end{pmatrix}$ ,  $\begin{pmatrix} 3 \\ 10 \\ 1 \\ 10 \end{pmatrix}$ ,  $\begin{pmatrix} 1 \\ 3 \\ h \\ h+3 \end{pmatrix}$  are linearly dependent.

**Solution:**

3. Give an example of two linear transformations  $S$  and  $T$  from  $\mathbb{R}^2$  to  $\mathbb{R}^2$  such that  $S \circ T \neq T \circ S$ . Hint: Consider  $2 \times 2$  matrices.

**Solution:**

4. If it exists, calculate the inverse matrix of  $\begin{pmatrix} 1 & 0 & 1 & -1 \\ 1 & 1 & 4 & 5 \\ 1 & 1 & 3 & 7 \\ 1 & 0 & 1 & -2 \end{pmatrix}$ .

**Solution:**

5. Let  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$  be a linear transformation. Is the set

$$T \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}, T \begin{pmatrix} 3 \\ 5 \\ 0 \end{pmatrix}, T \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

linearly independent? Justify your answer. Hint:  $T(\mathbf{0}) = \mathbf{0}$ .

**Solution:**