

MATH 1B MIDTERM 1 (PRACTICE 1)
PROFESSOR PAULIN

**DO NOT TURN OVER UNTIL
INSTRUCTED TO DO SO.**

CALCULATORS ARE NOT PERMITTED

**THIS EXAM WILL BE ELECTRONICALLY
SCANNED. MAKE SURE YOU WRITE ALL
SOLUTIONS IN THE SPACES PROVIDED.
YOU MAY WRITE SOLUTIONS ON THE
BLANK PAGE AT THE BACK BUT BE
SURE TO CLEARLY LABEL THEM**

Formulae

$$\begin{aligned} \int \tan(x) dx &= \ln |\sec(x)| + C & \int \sec(x) dx &= \ln |\sec(x) + \tan(x)| + C \\ \int \frac{1}{1+x^2} dx &= \arctan(x) + C & \int \frac{1}{\sqrt{1-x^2}} dx &= \arcsin(x) + C \\ \frac{d \tan(x)}{dx} &= \sec^2(x) & \frac{d \sec(x)}{dx} &= \tan(x) \sec(x) \\ 1 &= \sin^2(x) + \cos^2(x) & 1 + \tan^2(x) &= \sec^2(x) \\ \cos^2(x) &= \frac{1 + \cos(2x)}{2} & \sin^2(x) &= \frac{1 - \cos(2x)}{2} \\ |E_{Mid_n}| &\leq \frac{K(b-a)^3}{24n^2} & |E_{S_n}| &\leq \frac{K(b-a)^5}{180n^4} \end{aligned}$$

Name: _____

Student ID: _____

GSI's name: _____

This exam consists of 5 questions. Answer the questions in the spaces provided.

1. Compute the following integrals:

(a) (10 points)

$$\int x \sec^2(x) dx$$

Solution:

(b) (15 points)

$$\int \frac{1}{\sqrt{9x^2 - 1}} dx$$

Solution:

2. (a) (15 points) Express the following rational function

$$\frac{2x^2 + 4x + 1}{x^3 + 2x^2 + x}$$

as a sum of partial fractions.

Solution:

- (b) (10 points) Hence evaluate the integral

$$\int \frac{2x^2 + 4x + 1}{x^3 + 2x^2 + x} dx$$

Solution:

3. (25 points) Find the arc length of the curve

$$y = \frac{(x+1)^3}{3} + \frac{1}{4x+4}$$

between $x = 0$ and $x = 1$.

Solution:

4. Determine if the following improper integrals are convergent or divergent. Justify your answers.

(a) (10 points)

$$\int_0^{\infty} \frac{4 + \sin(x)}{e^{3x}} dx$$

Solution:

(b) (15 points)

$$\int_0^5 \frac{2x}{x^2 - 1} dx$$

5. (a) (15 points) For n a positive integer, let Mid_n be the midpoint approximation of the definite integral

$$\int_0^1 e^{2x^2} dx.$$

How large do we need to choose n to be to guarantee that the estimate is within 0.01 of the true value? You do not need to give an exact answer, just a carefully justified bound.

Solution:

- (b) (10 points) Is this approximation an overestimate or an underestimate? Be sure to justify your answer.

Solution: