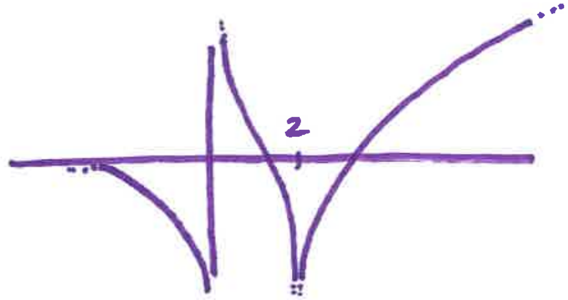
1A HOMEWORK 5 SOLUTIONS

§2.6/

Q7/



Q14/ $\displaystyle \lim_{x \to -\infty} \sqrt{\frac{9x^3 + 8x - 4}{3 - 5x + x^3}} = \lim_{x \to -\infty} \sqrt{\frac{9 + 8/x^2 - 4/x^3}{\frac{3}{x^3} - \frac{5}{x^2} + 1}} = \sqrt{\lim_{x \to -\infty} \frac{9 + 8/x^2 - 4/x^3}{\frac{3}{x^3} - \frac{5}{x^2} + 1}}$

$\displaystyle = \sqrt{\frac{9}{1}} = 3$

Q14/ $\displaystyle \lim_{t \to \infty} \frac{\sqrt{t} + t^2}{2t - t^2} = \lim_{t \to \infty} \frac{\frac{1}{t^{3/2}} + 1}{\frac{2}{t} - 1} = \frac{1}{-1} = -1$

Q24/ $\displaystyle \lim_{x \to -\infty} \frac{\sqrt{1 + 4x^6}}{2 - x^3} = \lim_{x \to -\infty} \frac{\left(\frac{\sqrt{1 + 4x^6}}{-\sqrt{x^6}}\right)}{\frac{2}{x^3} - 1} = \lim_{x \to -\infty} \frac{-\sqrt{\frac{1}{x^6} + 4}}{\frac{2}{x^3} - 1}$

$\displaystyle = \frac{-\sqrt{4}}{-1} = 2$

Q32/ $\displaystyle \lim_{\substack{x \to a \\ a^+ \\ a^- \\ \infty \\ -\infty}} f(x)$ exist <u>and</u> $\displaystyle \lim_{\substack{x \to a \\ a^+ \\ a^- \\ \infty \\ -\infty}} g(x)$ DNE $\Rightarrow$ $\displaystyle \lim_{\substack{x \to a \\ a^+ \\ a^- \\ \infty \\ -\infty}} (f(x) + g(x))$ DNE

$\lim_{x \to \infty} e^{-x} = 0$ <u>and</u> $\lim_{x \to \infty} 2\cos(3x)$ DNE $\Rightarrow \lim_{x \to \infty} \left(e^{-x} + 2\cos(3x)\right)$ DNE

Q40/ $\lim_{x \to 0^+} \ln(x) = -\infty$, $\lim_{u \to -\infty} \arctan(u) = \frac{-\pi}{2}$

$\Rightarrow \lim_{x \to 0^+} \arctan(\ln(x)) = \frac{-\pi}{2}$

Q42/ $\lim_{x \to \infty} \left(\ln(2+x) - \ln(1+x)\right) = \lim_{x \to \infty} \ln\left(\frac{2+x}{1+x}\right)$

$\lim_{x \to \infty} \frac{2+x}{1+x} = \lim_{x \to \infty} \frac{\frac{2}{x}+1}{\frac{1}{x}+1} = \frac{1}{1} = 1$

($\ln(x)$ cts at 1)

$\Rightarrow \lim_{x \to \infty} \left(\ln(2+x) - \ln(1+x)\right) = \ln(1) = 0$

Q50/ $y = \frac{1+x^4}{x^2 - x^4} = \frac{1+x^4}{x^2(1-x)(1+x)}$

$\Rightarrow x = 0, -1, 1$ are vertical asymptotes

$\lim_{x \to \infty, -\infty} \frac{1+x^4}{x^2 - x^4} = \lim_{x \to \infty, -\infty} \frac{\frac{1}{x^4}+1}{\frac{1}{x^2}-1} = \frac{1}{-1} = -1 \Rightarrow y = -1$ horizontal asymptote.

§2.7/

Q5/ $$\frac{dy}{dx} = \lim_{h\to 0} \frac{4(x+h) - 3(x+h)^2 - (4x - 3(x^2))}{h}$$

$$= \lim_{h\to 0} \frac{4h - 6xh - 3h^2}{h} = \lim_{h\to 0} 4 - 6x - 3h = 4 - 6x$$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{x=2} = 4 - 12 = -8$$

$\Rightarrow$ Tangent is $y - (-4) = -8(x-2)$

Q8/ $$y = \frac{2x+1}{x+2} = \frac{2x+4-3}{x+2} = 2 - \frac{3}{x+2}$$

$$\Rightarrow \frac{dy}{dx} = \lim_{h\to 0} \frac{\left(2 - \frac{3}{x+h+2}\right) - \left(2 - \frac{3}{x+2}\right)}{h} = \lim_{h\to 0} \frac{\frac{3}{x+2} - \frac{3}{x+h+2}}{h}$$

$$= \lim_{h\to 0} \frac{3h}{h(x+2)(x+h+2)} = \lim_{h\to 0} \frac{3}{(x+2)(x+h+2)} = \frac{3}{(x+2)^2}$$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{x=1} = \frac{3}{(1+2)^2} = \frac{1}{3}$$

$\Rightarrow$ Tangent is $y - 1 = \frac{1}{3}(x-1)$.

Q13 $y = 40t - 16t^2 \Rightarrow \frac{dy}{dt} = \lim_{h\to 0} \frac{40(t+h) - 16(t+h)^2 - 40t + 16t^2}{h}$

$$= \lim_{h\to 0} \frac{40h - 32th - 16h^2}{h} = \lim_{h\to 0} 40 - 32t - 16h = 40 - 32t.$$

$$\Rightarrow \left.\frac{dy}{dt}\right|_{t=2} = -24 \text{ ft/s}.$$

Q34 $f'(a) = \lim_{h\to 0} \frac{\frac{1}{(a+h)^2} - \frac{1}{a^2}}{h} = \lim_{h\to 0} \frac{a^2 - (a+h)^2}{h(a+h)^2 a^2} = \lim_{h\to 0} \frac{-2ah - h^2}{h(a+h)^2 a^2}$

$$= \lim_{h\to 0} \frac{-2a - h}{(a+h)^2 a^2} = \frac{-2a}{a^4} = \frac{-2}{a^3}$$

Q36 $f'(a) = \lim_{h\to 0} \frac{\frac{4}{\sqrt{1-a-h}} - \frac{4}{\sqrt{1-a}}}{h} = 4\cdot\lim_{h\to 0} \frac{\sqrt{1-a} - \sqrt{1-a-h}}{h\sqrt{1-a-h}\sqrt{1-a}}$

$$= 4\cdot\lim_{h\to 0} \frac{(1-a) - (1-a-h)}{h\sqrt{1-a-h}\sqrt{1-a}\left(\sqrt{1-a}+\sqrt{1-a-h}\right)} = 4\cdot\lim_{h\to 0} \frac{1}{\sqrt{1-a-h}\sqrt{1-a}\left(\sqrt{1-a}+\sqrt{1-a-h}\right)}$$

$$= 4\cdot\frac{1}{2(1-a)^{3/2}} = \frac{2}{(1-a)^{3/2}}.$$

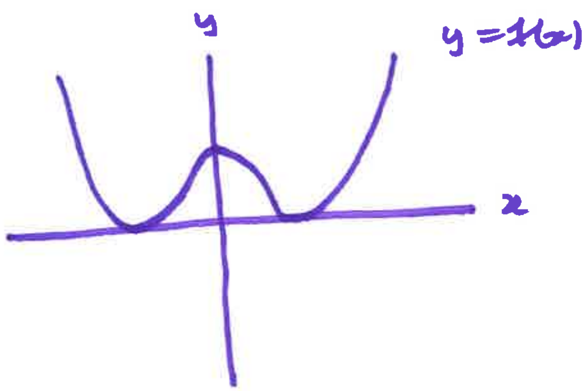
Q38/ $f(x) = e^x$, $a = -2$ $\Rightarrow f'(-2) = \lim_{h\to 0} \dfrac{e^{-2+h} - e^{-2}}{h}$.

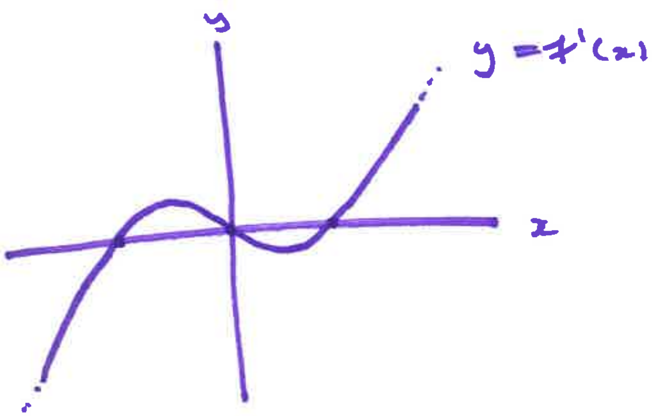
Q54/ $f'(0) = \lim_{h\to 0} \dfrac{h\sin\left(\frac{1}{h}\right) - 0}{h} = \lim_{h\to 0} \sin\left(\frac{1}{h}\right)$ DNE

Q60/ $f'(0) = \lim_{h\to 0} \dfrac{h^2\sin\left(\frac{1}{h}\right) - 0}{h} = \lim_{h\to 0} h\sin\left(\frac{1}{h}\right) \underset{\text{Squeeze}}{\overset{}{=}} 0$

§2-8/

Q4/





Q22/ $f(x) = mx+b \Rightarrow f'(x) = \lim_{h\to 0} \dfrac{m(x+h)+b-mx-b}{h}$

$= \lim_{h\to 0} \dfrac{mh}{h} = \lim_{h\to 0} m = m$

Domain of both $f(x)$ and $f'(x)$ is $\mathbb{R}$.

Q26/ $g'(t) = \lim_{h\to 0} \dfrac{\frac{1}{\sqrt{t+h}} - \frac{1}{\sqrt{t}}}{h}$

$= \lim_{h\to 0} \dfrac{\sqrt{t} - \sqrt{t+h}}{h\sqrt{t+h}\sqrt{t}} \cdot \dfrac{\sqrt{t}+\sqrt{t+h}}{\sqrt{t}+\sqrt{t+h}}$

$= \lim_{h\to 0} \dfrac{-h}{h\sqrt{t+h}\sqrt{t}\left(\sqrt{t}+\sqrt{t+h}\right)}$

$= \dfrac{-1}{2t^{3/2}}$ Both have domain $(0,\infty)$.

Q 28

$$f(x) = \frac{x^2-1}{2x-3} \Rightarrow f'(x) = \lim_{h\to 0} \frac{\dfrac{(x+h)^2-1}{2(x+h)-3} - \dfrac{x^2-1}{2x-3}}{h}$$

$$= \lim_{h\to 0} \frac{(2x-3)((x+h)^2-1) - (x^2-1)(2(x+h)-3)}{h\,(2(x+h)-3)(2x-3)} = \lim_{h\to 0} \frac{(2x-3)(2hx+h^2) - 2h(x^2-1)}{h(2(x+h)-3)(2x-3)}$$

$$= \lim_{h\to 0} \frac{(2x-3)(2x+h) - 2(x^2-1)}{(2(x+h)-3)(2x-3)} = \frac{(2x-3)\cdot 2x - 2(x^2-1)}{(2x-3)^2}.$$

Domain of both $f(x)$ and $f'(x)$ is all $\mathbb{R}$ minus $\frac{3}{2}$.

Q30

$$f(x) = x^{3/2} \Rightarrow f'(x) = \lim_{h\to 0} \frac{(x+h)^{3/2} - x^{3/2}}{h} \cdot \frac{(x+h)^{3/2} + x^{3/2}}{(x+h)^{3/2} + x^{3/2}}$$

$$= \lim_{h\to 0} \frac{(x+h)^3 - x^3}{h\,((x+h)^{3/2}+x^{3/2})} = \lim_{h\to 0} \frac{3hx^2+3h^2x+h^3}{h\,((x+h)^{3/2}+x^{3/2})} = \lim_{h\to 0} \frac{3x^2+3hx+h^2}{((x+h)^{3/2}+x^{3/2})}$$

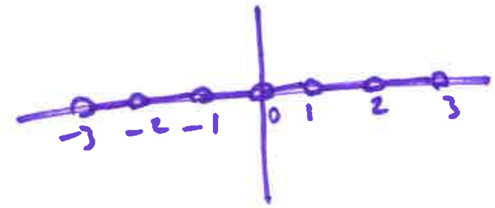
$$= \frac{3x^2}{2x^{3/2}} = \frac{3}{2}\sqrt{x}$$

Domain of $x^{3/2}$ is $[0, \infty)$. Domain of $f'(x)$ is $(0, \infty)$.

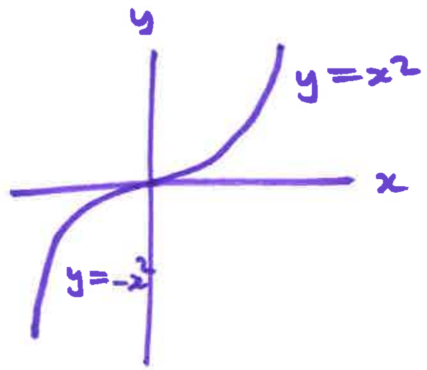
not differentiable at endpoint.

Q60 $f(x)$ not differentiable at k for k any integer.

$$f'(x) = \begin{cases} 0 & \text{if } x \text{ \underline{not} an integer} \\ \text{DNE} & \text{if } x \text{ an integer} \end{cases}$$



Q61 a) $f(x) = x|x| = \begin{cases} x^2 & \text{if } x \geq 0 \\ -x^2 & \text{if } x < 0 \end{cases}$



b) x^2 and $-x^2$ differentiable everywhere $\Rightarrow$ $f(x)$ certainly differentiable for $x \neq 0$

$$f'(0) = \lim_{h\to 0} \frac{h\cdot|h| - 0}{h} = \lim_{h \to 0} |h| = 0$$

$$\Rightarrow f'(x) = \begin{cases} 2x & \text{if } x \geq 0 \\ -2x & \text{if } x < 0 \end{cases} \qquad \left(\begin{array}{l} \frac{dx^2}{dx} = 2x \\ \frac{d(-x^2)}{dx} = -2x \end{array} \right)$$