Math 1A Homework 3 Solutions

§2.1

Q5/ a) i) -32 ft/s ii) -25.6 ft/s iii) -24.8 ft/s iv) -24.16 ft/s b) -24 ft/s

Q6/ a) i)
$$\frac{(10\cdot 2 - 1.86\cdot 2^2) - (10\cdot 1 - 1.86(1^2))}{2-1}$$

ii)
$$\frac{(10\cdot(1.5) - 1.86(1.5)^2) - (10\cdot 1 - 1.86(1^2))}{1.5-1}$$

iii)
$$\frac{(10\cdot(1.1) - 1.86(1.1)^2) - (10\cdot 1 - 1.86(1^2))}{1.1-1}$$

iv)
$$\frac{(10\cdot(1.01) - 1.86(1.01)^2) - (10\cdot 1 - 1.86(1^2))}{1.01-1}$$

v)
$$\frac{(10\cdot(1.001) - 1.86(1.001)^2) - (10\cdot 1 - 1.86(1^2))}{1.001-1}$$

USE A CALCULATOR

b) 6.28 m/s

Q7/ a) i) 29.3 ft/s ii) 32.7 ft/s iii) 45.6 ft/s iv) 48.75 ft/s

b) After looking at the above results it seams the slope is increasing. Thus I would estimate it is between 29.3 ft/s and 32.7 ft/s.

§2.2

Q4/ a) $\lim_{x\to 2^-} f(x) = 3$, b) $\lim_{x\to 2^+} f(x) = 1$, c) $\lim_{x\to 2} f(x)$ DNE as $\lim_{x\to 2^-} f(x) \neq \lim_{x\to 2^+} f(x)$

d) $f(2) = 3$, e) $\lim_{x\to 4} f(x) = 4$, f) DNE

Q5/ a) $\lim_{x\to 1} f(x) = 2$, b) $\lim_{x\to 3^-} f(x) = 1$, c) $\lim_{x\to 3^+} f(x) = 4$

d) $\lim_{x\to 3} f(x)$ DNE as $\lim_{x\to 3^-} f(x) \neq \lim_{x\to 3^+} f(x)$ e) $f(3) = 3$

Q6/ a) $\lim_{x\to -3^-} h(x) = 4$, b) $\lim_{x\to -3^+} h(x) = 4$, c) $\lim_{x\to -3} h(x) = 4$

d) $h(-3)$ DNE, e) $\lim_{x\to 0^-} h(x) = 1$, f) $\lim_{x\to 0^+} h(x) = -1$

g) $\lim_{x\to 0} h(x)$ DNE as $\lim_{x\to 0^-} h(x) \neq \lim_{x\to 0^+} h(x)$, h) $h(0) = 1$

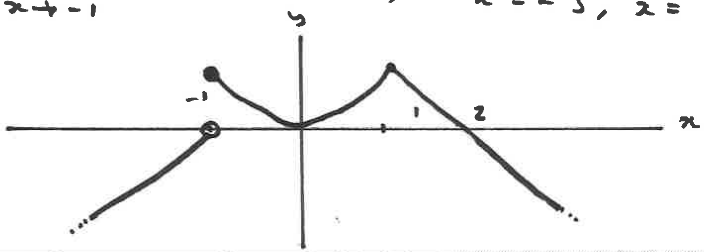
i) $\lim_{x\to 2} h(x) = 2$, j) $h(2)$ DNE, k) $\lim_{x\to 5^+} h(x) = 3$

l) $\lim_{x\to 5^-} h(x)$ DNE because it oscilates between 2 and 4.

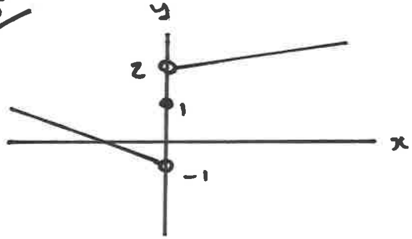
Q8/ a) $\lim_{x\to -3} A(x) = \infty$, b) $\lim_{x\to 2^-} A(x) = -\infty$, c) $\lim_{x\to 2^+} A(x) = \infty$

d) $\lim_{x\to -1} A(x) = -\infty$ e) $x = -3$, $x = -1$, $x = 2$

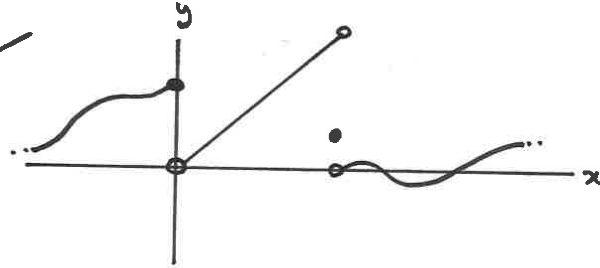
Q11/



Q15/



Q18/



Q31/ $\lim_{x\to 5^+} x+1 = 6$, $\lim_{x\to 5^+} (x-5) = 0$, Note that $x-5 \to 0^+$ as $x \to 5^+$

"approachs 0 positively"

$$\Rightarrow \lim_{x\to 5^+} \frac{x+1}{x-5} = \infty$$

Q34/

$$\lim_{x\to 3^-} \sqrt{x} = \sqrt{\lim_{x\to 3^-} x} = \sqrt{3} \neq 0$$

$(x-3)^5 \to 0^-$ as $x \to 3^-$

"approachs 0 negatively"

$$\Rightarrow \lim_{x\to 3^-} \frac{\sqrt{x}}{(x-3)^5} = -\infty .$$

Q37/

$\frac{1}{x}\sec(x) = \frac{1}{x\cos(x)}$, $x\cos(x) \to 0^-$ as $x \to \pi/2^+$

$$\Rightarrow \lim_{x\to \pi/2^+} \frac{1}{x}\sec(x) = -\infty$$

Q41/

$$\frac{x^2-2x-8}{x^2-5x+6} = \frac{(x-4)(x+2)}{(x-3)(x-2)}$$

$x-4 \to -2 < 0$ as $x \to 2^+$

$x+2 \to 4 > 0$ as $x \to 2^+$

$x-3 \to -1 < 0$ as $x \to 2^+$

$x-2 \to 0^+$ as $x \to 2^+$

$$\Rightarrow \lim_{x\to 2^+} \frac{x^2-2x-8}{x^2-5x+6} = \infty$$

Q42/ $\lim_{x \to 0^+} \frac{1}{x} = \infty$, $\lim_{x \to 0^+} (-\ln(x)) = \infty$

$$\Rightarrow \lim_{x \to 0^+} \left(\frac{1}{x} - \ln(x)\right) = \infty$$

Q52/

a) $f\left(\frac{1}{\left(\frac{1}{k\pi}\right)}\right) = \tan(k\pi) = 0$ for all $k \geq 1$ an integer.

b) $f\left(\frac{1}{\frac{4}{(4k+1)\pi}}\right) = f\left(\frac{(4k+1)\pi}{4}\right) = \tan\left(\frac{\pi}{4} + k\pi\right) = 1$

c) $\lim_{x \to 0^+} \tan\left(\frac{1}{x}\right)$ DNE as we have two sequences $\left\{\frac{1}{k\pi}\right\}$ and $\left\{\frac{4}{(4k+1)\pi}\right\}$ both of which tend to 0 as k increases, but $f(x)$ takes value 0 at the former and 1 at the latter.

§2.3

Q2/ a) $\lim_{x\to 2}(f(x)+g(x)) = \lim_{x\to 2} f(x) + \lim_{x\to 2} g(x) = -1+2 = 1$

b) $\lim_{x\to 2}(f(x)-g(x)) = -1-2 = -3$

c) $\lim_{x\to -1}(f(x)g(x)) = \lim_{x\to -1} f(x) \cdot \lim_{x\to -1} g(x) = 1 \cdot 2 = 2$

d) $\lim_{x\to 3} g(x) = 0$, $\lim_{x\to 3} f(x) = 1 \neq 0 \Rightarrow \lim_{x\to 3} \frac{f(x)}{g(x)}$ DNE

e) $\lim_{x\to 2}(x^2 f(x)) = \lim_{x\to 2} x^2 \cdot \lim_{x\to 2} f(x) = 2^2 \cdot (-1) = -4$

f) $f(-1) + \lim_{x\to -1} g(x) = 3+2 = 5$

Q3/ $\lim_{x\to 3}(5x^3 - 3x^2 + x - 6) = 5 \cdot 27 - 27 + 3 - 6 = 105$ (Limits at polynomials)

Q4/ $\lim_{x\to 2} 2x^2+1 = 9$, $\lim_{x\to 2} 3x-2 = 4 \neq 0$ (Polynomials)

$\Rightarrow \lim_{x\to 2} \frac{2x^2+1}{3x-2} = \frac{9}{4}$ (Quotient Law)

$\Rightarrow \lim_{x\to 2} \sqrt{\frac{2x^2+1}{3x-2}} = \sqrt{\frac{9}{4}} = \frac{3}{2}$ (Power Law)

Q10/ a) The functions are not equal for all x. The first is not defined at $x=2$, while the second is.

b) $$\frac{x^2+x-6}{x-2} = \frac{(x-2)(x+3)}{(x-2)} = x+3 \quad \text{if} \quad x \neq 2$$

$$\Rightarrow \lim_{x \to 2} \frac{x^2+x-6}{x-2} = \lim_{x \to 2} x+3 .$$

Q16/ $$\frac{2x^2+3x+1}{x^2-2x-3} = \frac{(2x+1)(x+1)}{(x-3)(x+1)} = \frac{2x+1}{x-3} \quad \text{if} \quad x \neq -1$$

$$\Rightarrow \lim_{x \to -1} \frac{2x^2+3x+1}{x^2-2x-3} = \lim_{x \to -1} \frac{2x+1}{x-3} = \frac{\lim_{x \to -1} 2x+1}{\lim_{x \to -1} x-3} = \frac{-1}{-4} = \frac{1}{4}$$

Q18/ $$\frac{(2+h)^3-8}{h} = \frac{\cancel{2^3} + 3 \cdot 2^2 \cdot h + 3 \cdot 2 \cdot h^2 - \cancel{2^3}}{h}$$

$$= 12 + 6h$$

$$\Rightarrow \lim_{h \to 0} \frac{(2+h)^3-8}{h} = \lim_{h \to 0} (12+6h) = 12$$

Q23/ $$\frac{\frac{1}{x} - \frac{1}{3}}{x-3} = \frac{\left(\frac{3-x}{3x}\right)}{x-3} = \frac{-1}{3x} \quad \text{if} \quad x \neq 3$$

$$\Rightarrow \lim_{x \to 3} \frac{\frac{1}{x} - \frac{1}{3}}{x-3} = \lim_{x \to 3} \frac{-1}{3x} = \frac{\lim_{x \to 3} (-1)}{\lim_{x \to 3} 3x} = \frac{-1}{9}$$

Q32

$$\frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} = \frac{x^2 - (x+h)^2}{h(x+h)^2 x^2} = \frac{-2xh - h^2}{h(x+h)^2x^2} \underset{h \neq 0}{=} \frac{-2x-h}{(x+h)^2x^2}$$

$$\Rightarrow \lim_{h \to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} = \lim_{h \to 0} \frac{-2x-h}{(x+h)^2x^2} = \frac{\lim_{h \to 0}(-2x-h)}{\lim_{h \to 0}(x+h)^2x^2} = \frac{-2x}{x^4} = \frac{-2}{x^3}.$$

Q39

$$x \neq 0 \Rightarrow -1 \leq \cos\left(\frac{2}{x}\right) \leq 1 \Rightarrow -x^4 \leq x^4 \cos\left(\frac{2}{x}\right) \leq x^4$$

$$\lim_{x \to 0} -x^4 = \lim_{x \to 0} x^4 = 0 \underset{\text{Squeeze}}{\Rightarrow} \lim_{x \to 0} x^4 \cos\left(\frac{2}{x}\right) = 0$$

Q45

$$x < 0 \Rightarrow |x| = -x \Rightarrow \frac{1}{x} - \frac{1}{|x|} = \frac{2}{x} \Rightarrow$$

$$\lim_{x \to 0^-} \frac{1}{x} - \frac{1}{|x|} = -\infty$$

Q46

$$x > 0 \Rightarrow |x| = x \Rightarrow \frac{1}{x} - \frac{1}{|x|} = 0 \Rightarrow \lim_{x \to 0^+}\left(\frac{1}{x} - \frac{1}{|x|}\right) = 0$$