MATH 1A HOMEWORK 2 SOLUTIONS

§1.3

Q2/ a) Shifting it up by 8, b) Shifting to left by 8, c) Vertically stretching by 8, d) Horizontally shrinking by 8, e) Reflecting in x-axis and shifting down by 1, f) Stretching both vertically and horizontally by 8

Q3/ a) 3, b) 1, c) 4, d) 5, e) 2

Q6/ Must shift to right by 2, then vertically stretch by 2 $\Rightarrow$

$$y = 2\sqrt{3(x-2)-(x-2)^2}$$

Q7/ Must shift to left by 4, reflect in x-axis and shift down by 1 $\Rightarrow$

$$y = -\sqrt{3(x+4)-(x+4)^2} - 1$$

Q31/ a) $f(x)+g(x) = x^3+5x^2-1$, Domain $= \mathbb{R}$

b) $f(x)-g(x) = x^3-x^2+1$, Domain $= \mathbb{R}$

c) $f(x)g(x) = 3x^5+6x^4-x^3-2x^2$, Domain $= \mathbb{R}$

d) $\frac{f(x)}{g(x)} = \frac{x^3+2x^2}{3x^2-1}$. $3x^2-1=0 \Leftrightarrow x = \pm\frac{1}{\sqrt{3}}$ $\Rightarrow$ Domain is all $\mathbb{R}$ except $\left\{\pm\frac{1}{\sqrt{3}}\right\}$.

Q32/ a) $f(x)+g(x) = \sqrt{3-x} + \sqrt{x^2-1}$

$3-x \geq 0 \Leftrightarrow 3 \geq x$

$x^2-1 \geq 0 \Leftrightarrow |x| \geq 1$

$\Rightarrow$ Domain is $(-\infty,-1] \cup [1,3]$. ("union")

b) $f(x) - g(x) = \sqrt{3-x} - \sqrt{x^2-1}$ Domain $= (-\infty, -1] \cup [1, 3]$

c) $f(x)\,g(x) = \sqrt{3-x} \cdot \sqrt{x^2-1}$ Domain $= (-\infty, -1] \cup [1, 3]$

d) $\dfrac{f(x)}{g(x)} = \dfrac{\sqrt{3-x}}{\sqrt{x^2-1}}$ Domain $= (-\infty, -1) \cup (1, 3]$ (not including ± 1)

Q35/ a) $f \circ g(x) = \sqrt{(4x-3)+1}$

$4x - 2 \geq 0 \iff x \geq \frac{1}{2}$

$\Rightarrow$ Domain $= [\frac{1}{2}, \infty)$

b) $g \circ f(x) = 4\sqrt{x+1} - 3$

$x + 1 \geq 0 \iff x \geq -1$

$\Rightarrow$ Domain $= [-1, \infty)$

c) $f \circ f(x) = \sqrt{\sqrt{x+1} + 1}$

$x + 1 \geq 0 \iff x \geq -1$

$\sqrt{x+1} + 1 \geq 0$ for all x where $\sqrt{x+1}$ defined

$\Rightarrow$ Domain $= [-1, \infty)$

d) $g \circ g(x) = 4(4x-3) - 3$, Domain $= \mathbb{R}$

Q37/ a) $f \circ g(x) = \dfrac{x+1}{x+2} + \dfrac{1}{(\frac{x+1}{x+2})}$, Domain is $\mathbb{R}$ minus $\{-1, -2\}$

b) $g \circ f(x) = \dfrac{(x + \frac{1}{x}) + 1}{(x + \frac{1}{x}) + 2}$, $x \neq 0 \Rightarrow g \circ f(x) = \dfrac{x^2 + 1 + x}{x^2 + 1 + 2x} = \dfrac{x^2 + x + 1}{(x+1)^2}$

$\Rightarrow$ Domain = $\mathbb{R}$ minus $\{0,-1\}$

c) $f(f(x)) = x + \frac{1}{x} + \frac{1}{x+\frac{1}{x}}$. $x \neq 0 \Rightarrow \frac{1}{x+\frac{1}{x}} = \frac{x}{x^2+1}$

$x^2+1 \neq 0$ for all x

$\Rightarrow$ Domain = $\mathbb{R}$ minus $\{0\}$.

d) $g(g(x)) = \dfrac{\left(\frac{x+1}{x+2}\right)+1}{\left(\frac{x+1}{x+2}\right)+2}$, $x+2 \neq 0 \Rightarrow x \neq -2 \Rightarrow \frac{x+1}{x+2}+2 = \frac{3x+5}{x+2}$

$3x+5 = 0 \Leftrightarrow x = \frac{-5}{3}$

$\Rightarrow$ Domain = $\mathbb{R}$ minus $\left\{-2, \frac{-5}{3}\right\}$

Q38/a) $f \circ g(x) = \dfrac{\sin(2x)}{1+\sin(2x)}$. $1+\sin(2x) \neq 0 \Leftrightarrow \sin(2x) = -1 \Leftrightarrow 2x = \frac{3\pi}{2} + 2k\pi$ (integer)

$\Leftrightarrow x = \frac{3\pi}{4} + k\pi$

$\Rightarrow$ Domain = $\mathbb{R}$ minus $\frac{3\pi}{4} + k\pi$, k integer.

b) $g \circ f(x) = \sin\left(\frac{2x}{1+x}\right)$, Domain = $\mathbb{R}$ minus $\{-1\}$.

c) $f \circ f(x) = \dfrac{\frac{x}{1+x}}{1+\frac{x}{1+x}}$. $1+x \neq 0 \Rightarrow 1+\frac{x}{1+x} = \frac{1+2x}{1+x}$

$1+2x = 0 \Leftrightarrow x = \frac{-1}{2}$

$\Rightarrow$ Domain = $\mathbb{R}$ minus $\left\{-1, \frac{-1}{2}\right\}$

d) $g \circ g(x) = \sin(2\sin(2x))$, Domain = $\mathbb{R}$.

Q42/ $f \circ g \circ h\,(x) = \tan\left(\dfrac{\sqrt[3]{x}}{\sqrt[3]{x}-1}\right)$

Q51/ $h(t) = \cos(t)$

$g(t) = \sin(t)$

$f(t) = t^2$

Q53/ a) 4 , b) 3 , c) 0 , d) Does <u>not</u> exist $f(c) = 6$ not in domain of g.

e) 4 , f) -2.

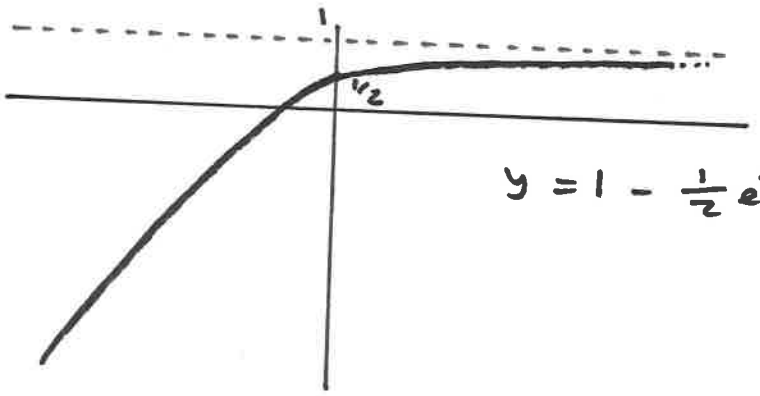
§1.4

Q2/ a) $8^{4/3} = (8^{\frac{1}{3}})^4 = 2^4 = 16$

b) $x(3x^2)^3 = 27x^7$

Q4/ a) $\dfrac{x^{2n} \cdot x^{3n-1}}{x^{n+2}} = \dfrac{x^{5n-1}}{x^{n+2}} = x^{4n-1}$, b) $\dfrac{\sqrt{a\sqrt{b}}}{\sqrt[3]{ab}} = \dfrac{a^{\frac{1}{2}} \cdot b^{\frac{1}{4}}}{a^{\frac{1}{3}} b^{\frac{1}{3}}} = a^{\frac{1}{6}} b^{-\frac{1}{12}}$.

Q15/



$y = 1 - \frac{1}{2}e^{-x}$

Q17/ a) $y = e^x - 2$, b) $y = e^{x-2}$

c) $y = -e^x$, d) $y = e^{-x}$

e) $y = -e^{-x}$

Q19/

a) $1 - e^{1-x^2} = 0 \Leftrightarrow 1 - x^2 = 0 \Leftrightarrow x = \pm 1 \Rightarrow$ Domain is $\mathbb{R}$ minus $\{\pm 1\}$.

b) $e^{\cos(x)} \neq 0$ for all x in $\mathbb{R} \Rightarrow$ Domain is $\mathbb{R}$

Q20/ a) $10^t - 100 \geq 0 \Leftrightarrow t \geq 2 \Rightarrow$ Domain is $[2, \infty)$

b) Domain is $\mathbb{R}$

Q22/ $f(x) = Cb^x$.

$f(-1) = 3 \Rightarrow 3 = Cb^{-1} \Rightarrow C = 3b$

$f(1) = \frac{4}{3} \Rightarrow \frac{4}{3} = Cb$

$\Rightarrow b^2 = \frac{4}{9} \Rightarrow b = \pm\frac{2}{3}$, but $b > 0$ so $b = \frac{2}{3} \Rightarrow C = 2$

$\Rightarrow f(x) = 2\left(\frac{2}{3}\right)^x$

Q23/ $\dfrac{f(x+h) - f(x)}{h} = \dfrac{5^{x+h} - 5^x}{h} = 5^x\left(\dfrac{5^h - 1}{h}\right)$

§1.5/

Q3/ Not one-to-one

Q5/ Not one-to-one

Q6/ One-to-one

Q8/ Not one-to-one

Q11/ Not one-to-one

Q13/ Not one-to-one

Q18/ a) Passes horizontal Line Test b) Domain = $[-3, 3]$, Range = $[-1, 3]$

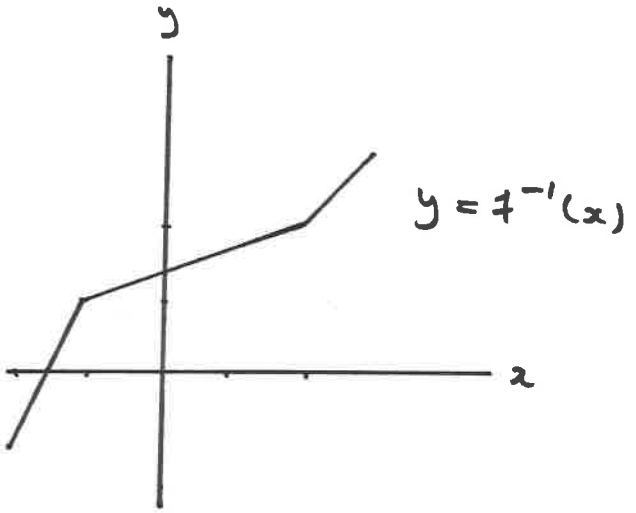
c) $f^{-1}(2) = 0$ d) $f^{-1}(0) = -\frac{3}{2}$

Q26/ Must solve $x = \dfrac{1-e^{-y}}{1+e^{-y}}$ in y.

$x = \dfrac{1-e^{-y}}{1+e^{-y}} \Rightarrow x + xe^{-y} = 1 - e^{-y} \Rightarrow \dfrac{x-1}{-x-1} = e^{-y}$

$\Rightarrow -y = \ln\left(\dfrac{x-1}{-x-1}\right) \Rightarrow y = -\ln\left(\dfrac{x-1}{-x-1}\right) = \ln\left(\dfrac{x+1}{1-x}\right)$

Q29/



Q36/ $\log_5\left(\frac{1}{125}\right) = \log_5\left(5^{-3}\right) = -3$.

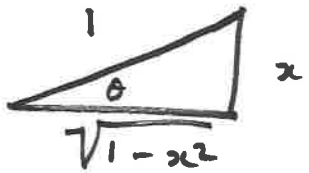
Q41/ $\frac{1}{3}\ln(x+2)^3 + \frac{1}{2}\left(\ln(x) - \ln(x^2+3x+2)^2\right)$

$= \ln\left(\dfrac{(x+2)}{\sqrt{x}\,|x^2+3x+2|}\right)$.

Q57/ $e^x - 3 > 0 \Leftrightarrow e^x > 3 \Leftrightarrow x > \ln(3)$ (ln increasing function)

$\Rightarrow$ Domain $= (\ln(3), \infty)$

Q69/ Let $\theta = \arcsin(x)$ $(\theta \geq 0)$ (ie $x \geq 0$) $\Rightarrow \sin\theta = x$



$\Rightarrow \cos(\arcsin(x)) = \cos\theta = \sqrt{1-x^2}$

If $x < 0 \Rightarrow \cos(\arcsin(x)) = \cos(-\arcsin(-x)) = \cos(\arcsin(-x)) = \sqrt{1-x^2}$ (arcsin odd)