Math 1A Homework 1 Solutions

Q1/ Yes, they are the same rule.

Q2/ Domain of $\frac{x^2-x}{x-1}$ is all numbers other than $x=1$

Domain of x is all real numbers

Different domains $\Rightarrow f \neq g$

Q3/ a) 3, b) -0.2, c) $0, 3$, d) -0.8, e) Domain $= [-2, 4]$ f) $[-2, 1]$
Range $= [-1, 3]$

Q4/ a) $f(-4) = -2$ b) $x = -2, 2$ c) $x = -3$ d) $[0, 4]$ e) Domain $= [-4, 4]$
Range $= [-2, 3]$
$g(3) = 4$

f) Domain $= [-4, 5]$
Range $= [\frac{1}{2}, 4]$

Q7/ No, fails vertical line test

Q10/ No, fails vertical line test

Q11/ a) 13.8 °C, b) 1990, c) 1910, 2005, d) $[13.5, 14.5]$

Q26/ $f(r) = ($Volume of sphere of radius $r+1)$ $-$ (Volume sphere of radius r) $= \frac{4}{3}\pi(r+1)^3 - \frac{4}{3}\pi r^3$

Q27/ $f(x) = 4 + 3x - x^2$

$$\Rightarrow \quad \frac{f(3+h)-f(3)}{h} = \frac{4+3(3+h)-(3+h)^2-4-3\cdot 3+3^2}{h}$$

$$= \frac{3h-6h-h^2}{h} = \frac{-3h-h^2}{h}$$

Q36/ $f(u) = \dfrac{u+1}{1+\frac{1}{u+1}} = \dfrac{u+1}{\left(\frac{u+2}{u+1}\right)}$

u in domain $\Leftrightarrow$ $u+1 \neq 0$ and $\frac{u+2}{u+1} \neq 0$

$\Leftrightarrow$ $u \neq -1$ or -2

Q37/ $F(p) = \sqrt{2-\sqrt{p}}$

p in domain $\Leftrightarrow$ $2-\sqrt{p} \geq 0$ and $p \geq 0$

$\Leftrightarrow$ $2 \geq \sqrt{p}$ and $p \geq 0$

$\Leftrightarrow$ $4 \geq p \geq 0$

Q51/ Slope $= \dfrac{7-(-3)}{5-1} = \dfrac{10}{4} \Rightarrow y = \frac{10}{4}x + c$

and $-3 = \frac{10}{4}\cdot 1 + c \Rightarrow c = \frac{-22}{4} \Rightarrow f(x) = \frac{10}{4}x - \frac{22}{4}$

Domain is $[1, 5]$.

Q58/

Area $= xy = 16 \Rightarrow y = \frac{16}{x}$

Perimeter $= 2x + 2y = 2x + 2 \cdot \frac{16}{x} = 2x + \frac{32}{x}$.

Q63/ Volume = Length × breadth × height

$= (20-2x) \times (12-2x) \times x$

$= (20-2x)(12-2x)x$

Q69/ g is <u>even</u> as it is same after reflecting in y-axis

f is <u>odd</u> as it is the same after rotating by π around $(0,0)$

Q70/ f is neither odd or even as it does <u>not</u> have either symmetry

g is even because it is the same after reflecting in y-axis.

Q79/ <u>Claim</u> f, g even $\Rightarrow f+g$ even

<u>Proof</u> Let x be in domain of both f and g. Then

$f(-x) = f(x)$ <u>and</u> $g(-x) = g(x)$. Hence

$(f+g)(-x) = f(-x) + g(-x) = f(x) + g(x) = (f+g)(x)$

$\Rightarrow f+g$ is even

"means proof is complete".

□

Claim f, g odd $\Rightarrow$ $f+g$ odd

Proof Let x be in domain of f and g. Then $f(-x) = -f(x)$ and $g(-x) = -g(x)$. Then $(f+g)(-x) = f(-x) + g(-x) = -f(x) - g(x)$

$= -(f+g)(x)$

Hence $f+g$ is odd. □

If f is even and g odd we cannot draw any conclusion about $f+g$. E.g. $f(x) = x^2$, $g(x) = x$ $\Rightarrow (f+g)(x) = x^2 + x$

$(f+g)(-1) = (-1)^2 + (-1) = 0$; $(f+g)(1) = 1^2 + 1 = 2$

$\Rightarrow$ $f+g$ neither even or odd.

§1.2/

Q3/ a) $y = \overset{x^2}{\overset{\shortparallel}{h(x)}}$ (classic parabola)

b) $y = x^5 = f(x)$ (negative as x goes below 0)

c) $y = x^8 = g(x)$ ($|x| < 1 \Rightarrow x^8 < x^2$ so similar to parabola but flatter near 0)

Q4/ a) $y = 3x = G(x)$ (straight line)

b) $y = 3^x = f(x)$ (exponential $3 > 1$)

c) $y = x^3 = F(x)$ (standard graph)

d) $y = \sqrt[3]{x} = g(x)$ (reflection of $y = x^3$ in $y = x$)

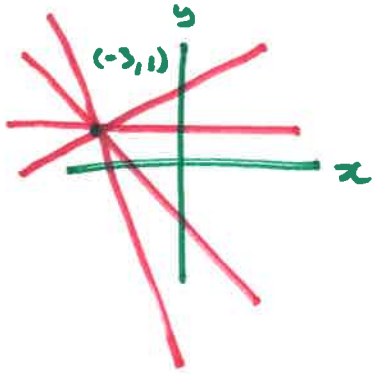
Q6/ x in domain $\Leftrightarrow$ $1 - \tan(x) \neq 0$ and $\tan(x)$ exists

$\Leftrightarrow$ $\tan(x) \neq 1$ and $\tan(x)$ exist

$\Leftrightarrow$ $x \neq \frac{\pi}{4} + n\pi$ where n is an integer.

and $x \neq \frac{\pi}{2} + n\pi$

Q8/ $y = f(x) = 1 + m(x+3)$ = straight line with slope m which contains point $(-3, 1)$



Q10/ i) $y = 2(x-3)^2$ (start with $y = x^2$, move to right by 3 and vertically stretch by 2)

ii) Let $y = ax^2 + bx + c$ $\Rightarrow$ $4a - 2b + c = 2$ $(-2, 2)$

$c = 1$ $(0, 1)$

$a + b + c = \frac{-5}{2}$ $(1, -\frac{5}{2})$

$\Rightarrow \quad \begin{matrix} 4a - 2b + 1 = 2 \\ a + b + 1 = \frac{-5}{2} \end{matrix} \Rightarrow \quad \begin{matrix} 2a - b + \frac{1}{2} = 1 \\ a + b + 1 = \frac{-5}{2} \end{matrix} \Rightarrow 3a + \frac{3}{2} = -\frac{3}{2}$

$\Rightarrow 3a = -3 \Rightarrow a = -1 \quad \Rightarrow \quad -4 - 2b + 1 = 2 \Rightarrow b = -\frac{5}{2}$

$\Rightarrow \quad f(x) = -x^2 - \frac{5}{2}x + 1.$

Q11/ $f(-1) = f(0) = f(2) = 0 \Rightarrow \quad f(x) = k(x+1)x(x-2)$

$f(1) = k \cdot 2 \cdot 1 \cdot -1 = -2k = 6 \Rightarrow k = -3$

$\Rightarrow f(x) = -3(x+1)x(x-2)$