# HW12 Solutions

## §6.1

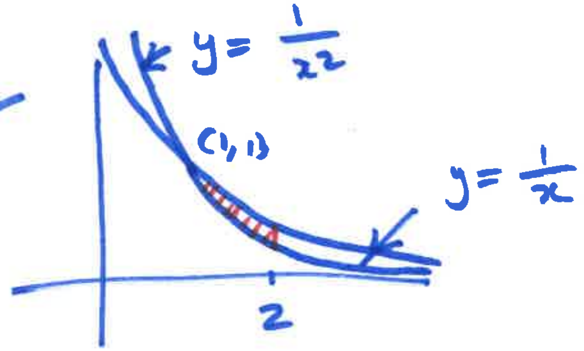
**Q1** Area $= \int_1^8 x^{1/3} - 1/x \, dx = \frac{3}{4}x^{4/3} - \ln|x| \Big|_1^8$

$= 12 - \ln(8) - \frac{3}{4} = \frac{45}{4} - \ln(8)$

**Q4** Area $= \int_0^3 (2y - y^2) - (y^2 - 4y)\, dy = \int_0^3 6y - 2y^2 \, dy$

$= 3y^2 - \frac{2}{3}y^3 \Big|_0^3 = 27 - 18 = 9$

**Q9**

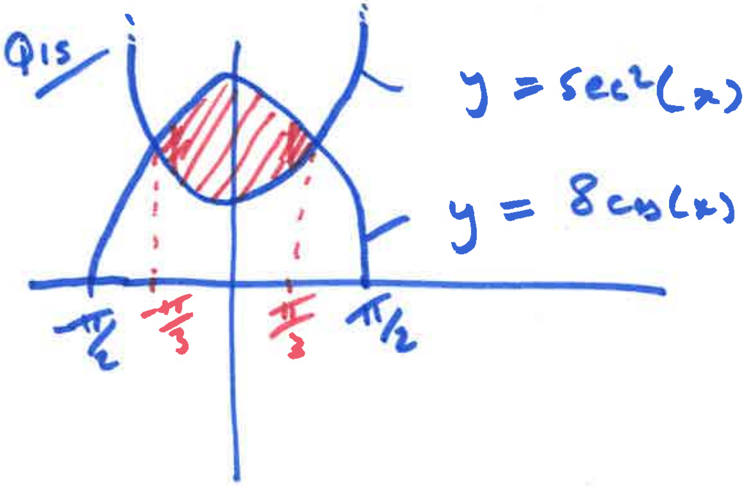



Area ( ) $= \int_1^2 1/x - 1/x^2 \, dx$

$= \ln|x| + \frac{1}{x} \Big|_1^2$

$= \ln(2) + \frac{1}{2} - 1 = \ln(2) - \frac{1}{2}$

Q1.

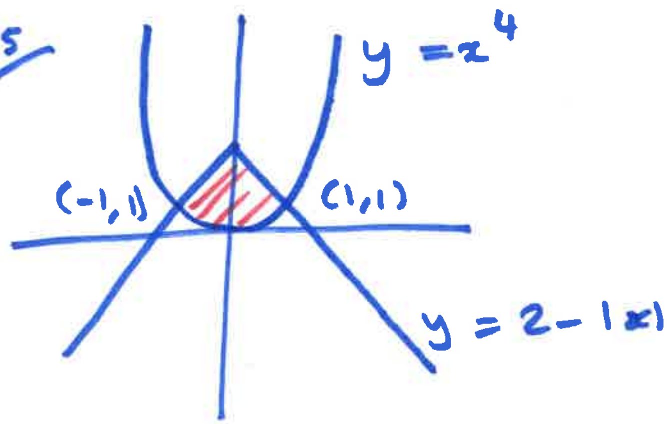



$$\text{Area} = \int_{-\pi/3}^{\pi/3} 8\cos(x) - \sec^2(x)\, dx$$

$$= 8\sin(x) - \tan(x) \Big|_{-\pi/3}^{\pi/3}$$

$$= 2\left(8\frac{\sqrt{3}}{2} - \sqrt{3}\right) = 8\sqrt{3} - 2\sqrt{3} = 6\sqrt{3}$$

Q2.




$$\text{Area} = \int_{-1}^{1} (2-|x|) - x^4\, dx$$

$$= \int_{0}^{1} 2 - x - x^4\, dx$$

$$+ \int_{-1}^{0} 2 + x - x^4\, dx$$

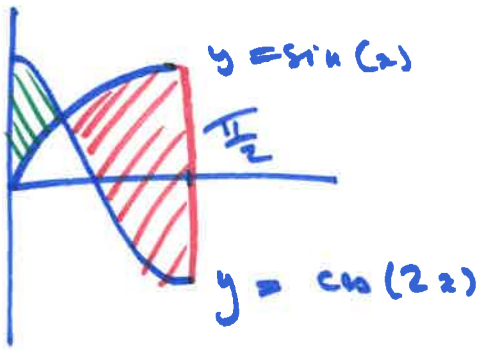
$$= 2x - \frac{1}{2}x^2 - \frac{1}{5}x^5 \Big|_0^1 + 2x + \frac{1}{2}x^2 - \frac{1}{5}x^5 \Big|_{-1}^{0}$$

$$= \left(2 - \frac{1}{2} - \frac{1}{5}\right) \times 2 = 4 - 1 - \frac{2}{5} = \frac{13}{5}$$

Q24 a) $12 + 27 = 39$

b) $27 - 12 = 15$

Q35




$$\sin(x) = \cos(2x) = \cos^2(x) - \sin^2(x)$$

$$= 1 - 2\sin^2(x)$$

$$\Rightarrow \quad 2\sin^2(x) + \sin(x) - 1 = 0$$

$$\Rightarrow \quad (2\sin(x) - 1)(\sin(x) + 1) = 0$$

$$\Rightarrow \quad \sin(x) = \frac{1}{2} \text{ or } \sin(x) = -1$$

$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$ on $\left[0, \frac{\pi}{2}\right]$ $\Rightarrow$ intersection is $\left(\frac{\pi}{6}, \frac{1}{2}\right)$

$$\text{Area } ( \; ) = \int_{\pi/6}^{\pi/2} \sin(x) - \cos(2x)\, dx = -\cos(x) - \frac{1}{2}\sin(2x) \Big|_{\pi/6}^{\pi/2}$$

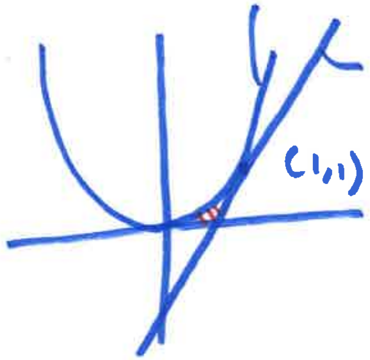
$$= \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{4} = \frac{3\sqrt{3}}{4}$$

$$\text{Area } ( \; ) = \int_0^{\pi/6} \cos(2x) - \sin(x)\, dx = \frac{1}{2}\sin(2x) + \cos(x) \Big|_0^{\pi/6}$$

$$= \frac{3\sqrt{3}}{4} - 1$$

$$\Rightarrow \int_0^{\pi/3} |\sin(x) - \cos(2x)|\, dx = \frac{3\sqrt{3}}{2} - 1$$

Q56




$y = x^2 \Rightarrow x = \sqrt{y}$

$y = 2x - 1$

$\Updownarrow$

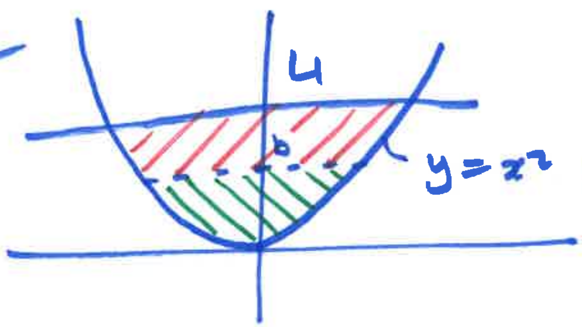
$x = \frac{1}{2}y + \frac{1}{2}$

$$\text{Area}(\ ) = \int_0^1 \frac{1}{2}y + \frac{1}{2} - y^{1/2}\, dy$$

$$= \frac{1}{4}y^2 + \frac{1}{2}y - \frac{2}{3}y^{3/2} \Big|_0^1$$

$$= \frac{1}{4} + \frac{1}{2} - \frac{2}{3} = \frac{1}{12}$$

Q57




$y = x^2 \Rightarrow x = \sqrt{y}$

$$\text{Area}(\ ) = 2\int_b^4 \sqrt{y}\, dy$$

$$= 2 \cdot \frac{2}{3} y^{3/2} \Big|_b^4$$

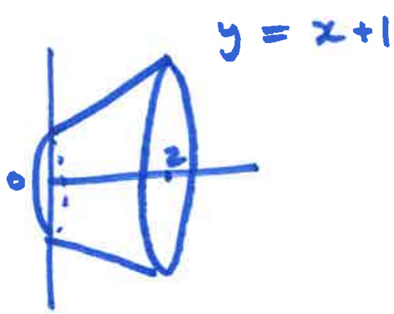
$$= \frac{32}{3} - \frac{4}{3} b^{3/2}$$

$$\text{Area}(\ ) = 2\int_0^b \sqrt{y}\, dy = 2\,\frac{2}{3} y^{3/2} \Big|_0^b = \frac{4}{3} b^{3/2}$$

$$\text{Area}(\ ) = \text{Area}(\ ) \Leftrightarrow \frac{32}{3} = \frac{8}{3} b^{3/2} \Leftrightarrow 4 = b^{3/2} \Leftrightarrow b = 4^{2/3}$$

§ 6.2

Q1

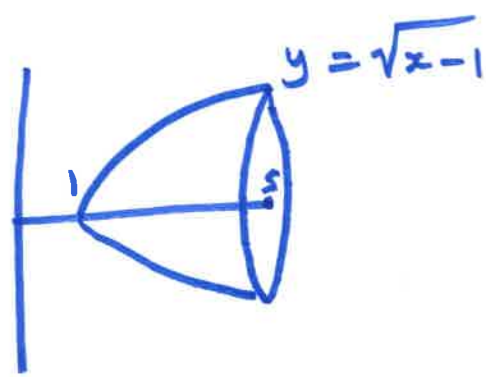



$$\Rightarrow \quad \text{Volume} = \int_0^2 \pi(x+1)^2\,dx = \pi\int_0^2 x^2+2x+1\,dx$$

$$= \pi\left(\tfrac{1}{3}x^3 + x^2 + x\right)\Big|_0^2$$

$$= \pi\left(\tfrac{8}{3} + 4 + 2\right) = \frac{26\pi}{3}$$

Q3

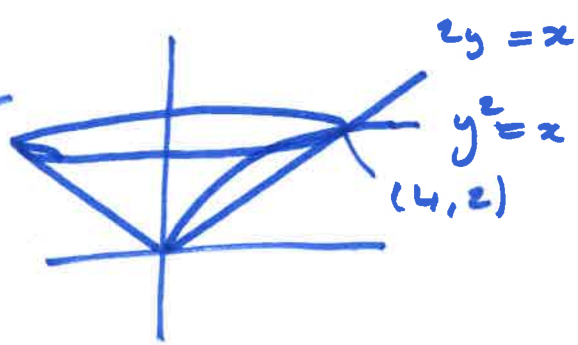



$$\text{Vol} = \int_1^5 \pi(x-1)\,dx$$

$$= \pi\left(\tfrac{1}{2}x^2 - x\right)\Big|_1^5$$

$$= \frac{25\pi}{2} - 5\pi - \frac{\pi}{2} + \pi = 8\pi$$

Q4




$$\text{Volume} = \int_0^2 \pi\left((2y)^2 - (y^2)^2\right)dy$$

$$= \pi\int_0^2 4y^2 - y^4\,dy = \frac{4\pi}{3}y^3 - \frac{\pi}{5}y^5\Big|_0^2$$

$$= \frac{32\pi}{3} - \frac{32\pi}{5} = \frac{64\pi}{15}$$

**Q14**

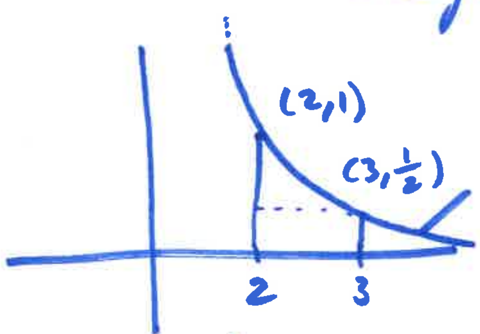
Volume $= \int_0^{\frac{\pi}{4}} \pi \left((\cos(x)+1)^2 - (\sin(x)+1)^2\right) dx$

$= \pi \int_0^{\pi/4} \cos(2x) + 2\cos(x) - 2\sin(x)\, dx$

$= \pi \left( \frac{1}{2}\sin(2x) + 2\sin(x) + 2\cos(x) \right) \Big|_0^{\pi/4}$

$= \frac{\pi}{2} + 2\sqrt{2}\pi - 2\pi = \left(2\sqrt{2} - \frac{3}{2}\right)\pi$

**Q16** Shift everything 1 to right

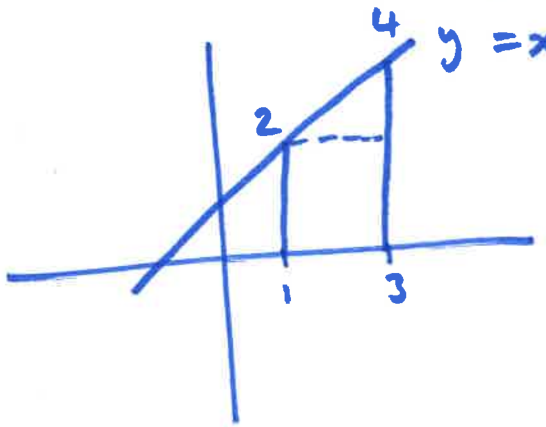



$y = \frac{1}{x-1} \Rightarrow x = \frac{1}{y} + 1$

Volume $= \int_0^{\frac{1}{2}} \pi(3^2 - 2^2)\, dy + \int_{\frac{1}{2}}^{1} \pi\left(\left(\frac{1}{y}+1\right)^2 - 2^2\right) dy$

$= 5\pi y \Big|_0^{\frac{1}{2}} + \pi\left(\frac{-1}{y} + 2\ln|y| - 3y\right)\Big|_{1/2}^{1} = \frac{5\pi}{2} + \pi\left(2\ln(2) - \frac{1}{2}\right)$

**Q18** Shift everything to left by 1




$y = x+1 \Rightarrow x = y-1$

$$\text{Volume} = \int_0^2 \pi (3^2 - 1^2)\, dy + \int_2^4 \pi (3^2 - (y-1)^2)\, dy$$

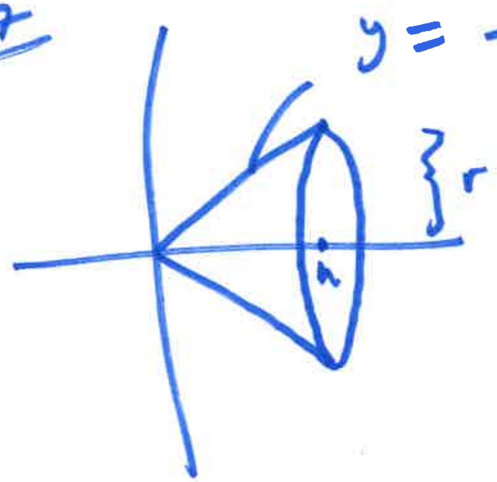
$$= 8\pi y \Big|_0^2 + \pi \left(8y + y^2 - \tfrac{1}{3}y^2\right)\Big|_2^4$$

$$= 16\pi + 24\pi = 40\pi.$$

**Q34** The solid is the region formed after revolving $y = \sqrt{\sin(x)}$ around the x-axis between 0 and $\pi$

**Q47**

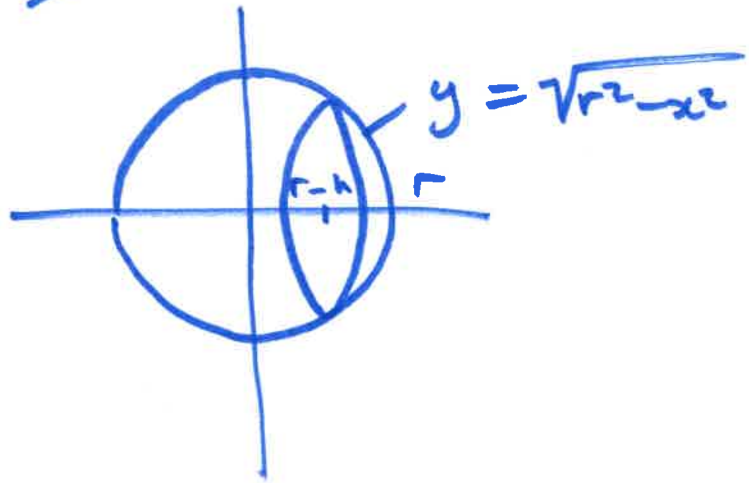
$y = \frac{r}{h} x$




$$\text{Volume} = \int_0^h \pi \frac{r^2}{h^2} x^2\, dx$$

$$= \frac{\pi r^2}{3h^2} x^3 \Big|_0^h = \frac{1}{3}\pi r^2 h.$$

Q49



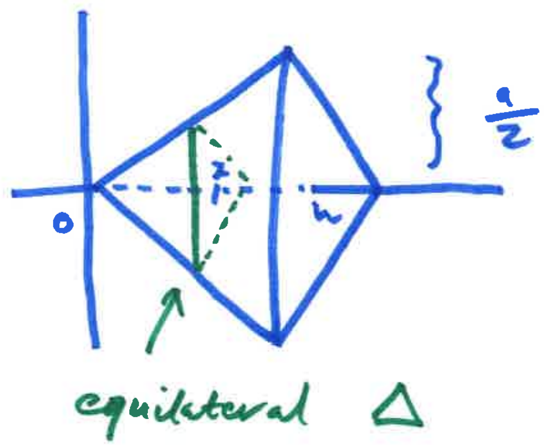

$$\text{Volume} = \int_{r-h}^{r} \pi r^2 - \pi x^2 \, dx$$

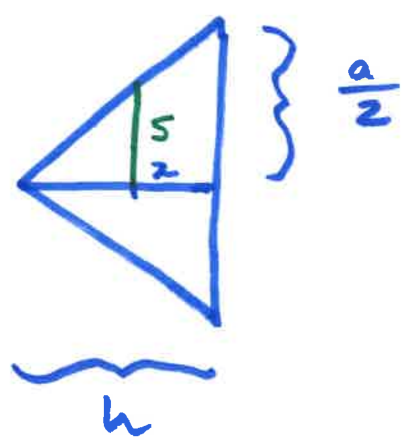
$$= \pi r^2 x - \frac{\pi}{3} x^3 \Big|_{r-h}^{r}$$

$$= \frac{2\pi}{3} r^3 - \pi r^2 (r-h) + \frac{\pi}{3}(r-h)^3$$

$$= \pi h^2 \left(r - \tfrac{1}{3} h\right)$$

Q52



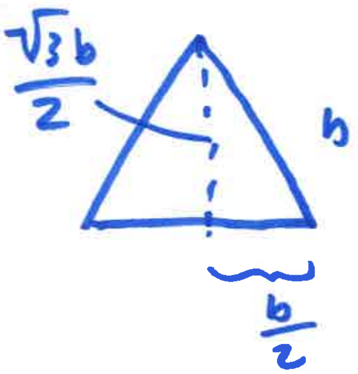




$$\Rightarrow \quad \frac{s}{x} = \frac{\frac{a}{2}}{h}$$

$$\Rightarrow \quad s = \frac{ax}{2h}$$

$$\Rightarrow \quad 2s = \frac{ax}{h}$$

$\Rightarrow$ Cross-section at $x$ is equilateral triangle with side $\frac{ax}{h}$

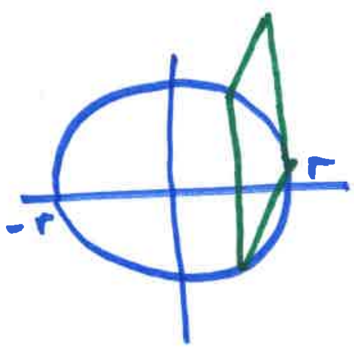



$\Rightarrow$ Area $= \frac{1}{2}\frac{\sqrt{3}}{2}b \cdot b = \frac{\sqrt{3}}{4}b^2$

$\Rightarrow A(x) = \frac{\sqrt{3}}{4}\frac{a^2x^2}{h^2}$

$\Rightarrow$ Volume $= \frac{\sqrt{3}a^2}{4h^2}\int_0^h x^2\,dx = \frac{\sqrt{3}\,a^2}{4h^2}\,\frac{1}{3}x^3\Big|_0^h = \frac{\sqrt{3}\,a^2h}{12}$.

Q54




$\Big\}\ 2\sqrt{r^2-x^2} \Rightarrow A(x) = 4r^2 - 4x^2$

$\Rightarrow$ Volume $= \int_{-r}^{r} 4r^2 - 4x^2\,dx = 4r^2x - \frac{4}{3}x^3\Big|_{-r}^{r}$

$= 2 \times \left(4r^3 - \frac{4}{3}r^3\right)$

$= \frac{16}{3}r^3$.