HW 11 Solution

§5.1

Q1/ Check Textbook

Q21/

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2\left(1+\frac{2i}{n}\right)}{\left(1+\frac{2i}{n}\right)^2+1} \frac{2}{n}$$

Q22/

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\left(4+\frac{3i}{n}\right)^2 + \sqrt{1+2\left(4+\frac{3i}{n}\right)}\right) \frac{3}{n}$$

Q24/ $a=1$, $b=4$, $f(x)=\sqrt{x}$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n} \sqrt{1+\frac{3i}{n}} = \int_1^4 \sqrt{x}\, dx$$

Q25/ $a=0$, $b=\frac{\pi}{4}$ $f(x)=\tan(x)$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\pi}{4n} \tan\frac{i\pi}{4n} = \int_0^{\pi/4} \tan(x)\, dx$$

§5.2

Q17 $\lim_{n \to \infty} \sum_{i=1}^{n} \frac{e^{x_i}}{1+x_i} \Delta x \,(, [0,1]) = \int_0^1 \frac{e^x}{1+x}\, dx$

Q18 $\lim_{n \to \infty} \sum_{i=1}^{n} x_i \sqrt{1+x_i^3}\, \Delta x \quad (\text{on } [2,5]) = \int_2^5 x\sqrt{1+x^3}\, dx$

Q19 $\lim_{n \to \infty} \sum_{i=1}^{n} (5(x_i^*)^3 - 4x_i^*)\, \Delta \quad (\text{on } [2,7]) = \int_2^7 (5x^3 - 4x)\, dx$

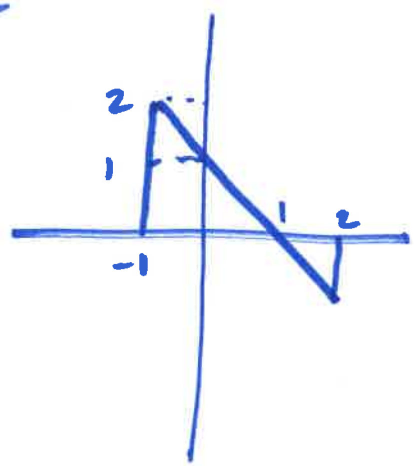
Q29 $\int_1^3 \sqrt{4+x^2}\, dx = \lim_{n \to \infty} \left(\sum_{i=1}^{n} \sqrt{4+(1+\frac{2i}{n})^2} \cdot \frac{2}{n} \right)$

Q30 $\int_2^5 x^2 + \frac{1}{x}\, dx = \lim_{n \to \infty} \left(\sum_{i=1}^{n} \left((2+\frac{3i}{n})^2 + \frac{1}{2+\frac{3i}{n}} \right) \frac{3}{n} \right)$

Q33 a) 4 , b) 10 , c) -3 , d) 2

Q34 a) 4 , b) -4π , c) $4 - 4\pi + \frac{1}{2}$.

Q35

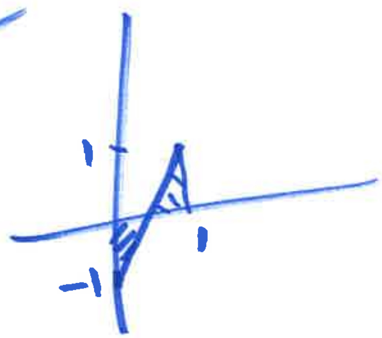


$$\int_{-1}^{2} 1 - x\,dx = 1 + \frac{1}{2} + \frac{1}{2} - \frac{1}{2} = \frac{3}{2}$$

Q38

$$\int_{-5}^{5} x - \sqrt{25 - x^2}\,dx = \underbrace{\int_{-5}^{5} x\,dx}_{0} - \underbrace{\int_{-5}^{5} \sqrt{25 - x^2}\,dx}_{-\frac{25\pi}{2}} = -\frac{25\pi}{2}$$

Q40



$$\Rightarrow \int_{0}^{1} |2x - 1|\,dx = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

Q59

$$\int_0^5 f(x)\,dx = \int_0^3 3\,dx + \int_3^5 x\,dx$$

$$= 3\times 3 + 3\times 2 + \frac{2\times 2}{2} = 17$$

Q57

$$1 \le \sqrt{1+x^2} \le \sqrt{2} \quad \text{for all } x \text{ in } [-1, 1]$$

$$\Rightarrow 2 \le \int_{-1}^{1} \sqrt{1-x^2}\,dx \le 2\sqrt{2}.$$

§5.3

Q4 a) $g(0) = 0$, $g(6) = 0$

b) $g(1) \approx 2.875$

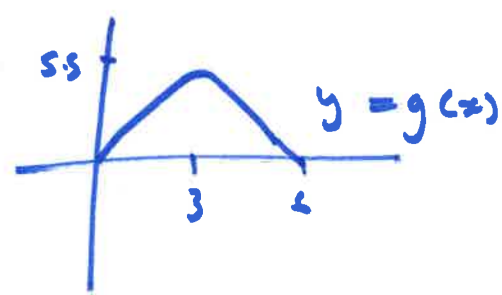
$g(2) \approx 4.75$

$g(3) \approx 5.5$

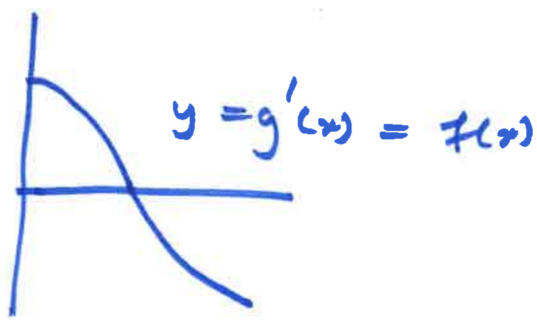
$g(4) \approx 4.75$

$g(5) \approx 2.875$

c) $[0, 3]$, d) $x = 3$, e)



f)



Q7 $g'(x) = \sqrt{x + x^3}$

Q12 $R(y) = -\int_2^y t^3 \sin(t)\,dt \Rightarrow R'(y) = -y^3 \sin(y)$

Q13 $A(x) = \int_1^x \ln(t)\,dt \Rightarrow A'(x) = \ln(x)$

$h(x) = A(e^x) \Rightarrow h'(x) = e^x A'(e^x) = e^x \ln(e^x)$

$= xe^x$

Q18 $A(x) = \int_1^x \sqrt{1+t^2}\,dt \Rightarrow A'(x) = \sqrt{1+x^2}$

$y = \int_{\sin(x)}^1 \sqrt{1+t^2}\,dt = -A(\sin(x)) \Rightarrow$

$\frac{dy}{dx} = -\cos(x)\,A'(\sin(x)) = -\cos(x)\sqrt{1+\sin^2(x)}$

Q23 $\int_0^1 1 - 8v^3 + 16v^7\,dv = v - 2v^4 + 2v^8 \Big|_0^1$

$= (1 - 2 + 2) - (0 - 0 + 0)$

$= 1$

Q24

$$\int_1^4 \frac{2+x^2}{\sqrt{x}}\,dx = \int_1^4 2 \cdot x^{-\frac{1}{2}} + x^{3/2}\,dx$$

$$= 2 \cdot \frac{1}{1-\frac{1}{2}} x^{\frac{1}{2}} + \frac{1}{1+\frac{3}{2}} x^{5/2} \Big|_1^4$$

$$= \frac{82}{5}.$$

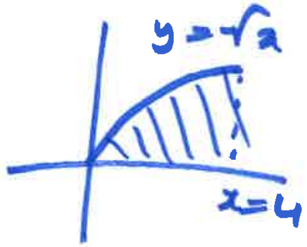
Q43

$$\int_0^{\pi} f(x)\,dx = \int_0^{\pi/2} \sin(x)\,dx + \int_{\pi/2}^{\pi} \cos(x)\,dx$$

$$= -\cos(x)\Big|_0^{\pi/2} + \sin(x)\Big|_{\pi/2}^{\pi}$$

$$= 1 - 1 = 0$$

Q45



$$\text{Area}(\,\|\|\,) = \int_0^4 \sqrt{x}\,dx = \frac{2}{3} x^{3/2}\Big|_0^4$$

$$= \frac{16}{3}.$$

Q63 $A(x) = \int_0^x \ln(1+2v)\,dv \Rightarrow A'(x) = \ln(1+2x)$

$$F(x) = \int_{\cos(x)}^{\sin(x)} \ln(1+2v)\,dv = \int_0^{\sin(x)} \ln(1+2v)\,dv - \int_0^{\cos(x)} \ln(1+2v)\,dv$$

$$= A(\sin(x)) - A(\cos(x))$$

$$\Rightarrow F'(x) = \cos(x)\,A'(\sin(x)) + \sin(x)\,A'(\cos(x))$$

$$= \cos(x)\ln(1+2\sin(x)) + \sin(x)\ln(1+2\cos(x)).$$

Q64 $17 = \int_1^4 F'(x)\,dx = F(4) - F(1) = F(4) - 12$

$$\Rightarrow F(4) = 17 + 12 = 29.$$

Q75 $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i^4}{n^5} + \frac{i}{n^2}\right) = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\left(\frac{i}{n}\right)^4 + \left(\frac{i}{n}\right)\right)\frac{1}{n}$

$\left(a = 0,\ b = 1,\ F(x) = x^4 + x,\ x_i^* = x_i = \frac{i}{n}\right)$

$$= \int_0^1 x^4 + x \, dx = \frac{1}{5}x^5 + \frac{1}{2}x^2 \Big|_0^1 = \frac{1}{5} + \frac{1}{2} = \frac{7}{10}$$

§5.4

Q7 $$\int 5 + \frac{2}{3}x^2 + \frac{3}{4}x^3 \, dx = 5x + \frac{2}{9}x^3 + \frac{3}{16}x^4 + C$$

Q11 $$\int \frac{1 + \sqrt{x} + x}{x} \, dx = \int \frac{1}{x} + x^{-\frac{1}{2}} + 1 \, dx$$

$$= \begin{cases} \ln(x) + 2\sqrt{x} + x + C_1 & \text{for } x > 0 \\ \ln(-x) + 2\sqrt{x} + x + C_2 & \text{for } x < 0 \end{cases}$$

Q15 $$\int 2 + \tan^2\theta \, d\theta = \int 2 + (\sec^2\theta - 1) \, d\theta$$

$$= \theta + \tan\theta + C$$

different constant for each open interval $\tan\theta$ is defined on.

Q25 $(2x-3)(4x^2+1) = 8x^3 - 12x^2 + 2x - 3$

$$\Rightarrow \int_0^2 (2x-3)(4x^2+1)\,dx = 2x^4 - 4x^3 + x^2 - 3x \Big|_0^2$$

$$= 32 - 32 + 4 - 6 = -2$$

Q33 $$\int_1^2 \left(\frac{x}{2} - \frac{2}{x}\right)dx = \frac{x^2}{4} - 2\ln(x)\Big|_1^2$$

$$= \left(1 - 2\ln(2)\right) - \left(\frac{1}{4}\right) = \frac{3}{4} - 2\ln(2)$$

Q46 $$\int_0^{3\pi/2} |\sin(x)|\,dx = \int_0^{\pi} \sin(x)\,dx - \int_{\pi}^{3\pi/2} \sin(x)\,dx$$

$$= -\cos(x)\Big|_0^{\pi} - \left(-\cos(x)\right)\Big|_{\pi}^{3\pi/2} = 2 - (-1)$$

$$= 3$$

Q51 $\int_5^{10} w'(t)\,dt = w(10) - w(5) =$ net change in child's weight between 5 and 10 years old.

Q61/

$a(t) = t+4 \Rightarrow V(t) = \frac{1}{2}t^2 + 4t + 5 \geqslant 0$ on $[0,10]$

$\Rightarrow$ Distance travelled between 0 and 10 $= \int_0^{10} \frac{1}{2}t^2 + 4t + 5 \, dt$

$= \frac{1}{6}t^3 + 2t^2 + 5t \Big|_0^{10} = 416\frac{2}{3}$ m

§5.5/

Q1/ $u = 2x \Rightarrow \frac{du}{dx} = 2 \Rightarrow dx = \frac{du}{2}$

$\Rightarrow \int \cos(2x)\,dx = \int \cos(2x)\frac{du}{2} = \frac{1}{2}\int \cos(u)\,du = \frac{1}{2}\sin(u) + C$

$= \frac{1}{2}\sin(2x) + C$

Q4/ $u = \sin\theta \Rightarrow \frac{du}{d\theta} = \cos\theta \Rightarrow d\theta = \frac{du}{\cos\theta} \Rightarrow$

$\int \sin^2\theta\cos\theta\,d\theta = \int \sin^2\theta\cos\theta\frac{du}{\cos\theta} = \int u^2\,du = \frac{1}{3}u^3 + C = \frac{1}{3}\sin^3\theta + C$

Q4/ $u = 1-2x \Rightarrow \frac{du}{dx} = -2 \Rightarrow dx = \frac{du}{-2} \Rightarrow$

$\int (1-2x)^9\,dx = \frac{1}{-2}\int u^9\,du = \frac{-1}{20}u^{10} + C = \frac{-1}{20}(1-2x)^{10} + C$

Q14/ $u = 3ax + bx^3 \Rightarrow \frac{du}{dx} = 3a + 3bx^2 \Rightarrow dx = \frac{du}{3a+3bx^2}$

$\Rightarrow \int \frac{a+bx^2}{\sqrt{3ax+bx^3}}\,dx = \frac{1}{3}\int \frac{1}{\sqrt{u}}\,du = \frac{2}{3}\sqrt{u} + C$

$= \frac{2}{3}\sqrt{3ax+bx^3} + C$

Q28/ $u = \cos(t) \Rightarrow \frac{du}{dt} = -\sin t \Rightarrow dt = \frac{du}{-\sin(t)} \Rightarrow$

$\int e^{\cos(t)}\sin(t)\,dt = -\int e^u\,du = -e^u + C = -e^{\cos(t)} + C$

Q48/ $u = x^2+1 \Rightarrow \frac{du}{dx} = 2x \Rightarrow dx = \frac{du}{2x} \Rightarrow$

$\int x^3\sqrt{x^2+1}\,dx = \frac{1}{2}\int x^2\sqrt{x^2+1}\,du = \frac{1}{2}\int (u-1)\sqrt{u}\,du$

$$= \frac{1}{2} \cdot \frac{1}{\left(\frac{5}{2}\right)} u^{5/2} - \frac{1}{2} \frac{1}{\frac{3}{2}} u^{3/2} + C$$

$$= \frac{1}{5}(x^2+1)^{5/2} - \frac{1}{3}(x^2+1)^{3/2} + C$$

Q60/ $u = -x^2 \Rightarrow \frac{du}{dx} = -2x \Rightarrow dx = \frac{du}{-2x} \Rightarrow$

$$\int x e^{-x^2} dx = \frac{-1}{2} \int e^u \, du = \frac{-1}{2} e^u + C = \frac{-1}{2} e^{-x^2} + C$$

$$\Rightarrow \int_0^1 x e^{-x^2} dx = \frac{-1}{2} e^{-1} + \frac{1}{2}.$$

Q65/ $u = x^2 + a^2 \Rightarrow \frac{du}{dx} = 2x \Rightarrow dx = \frac{du}{2x} \Rightarrow$

$$\int x \sqrt{x^2+a^2} \, dx = \frac{1}{2} \int \sqrt{u} \, du = \frac{1}{3} u^{3/2} + C$$

$$= \frac{1}{3}(x^2+a^2)^{3/2} + C \Rightarrow \int_0^a x\sqrt{x^2+a^2}\,dx = \frac{1}{3}(2a^2)^{3/2}.$$

Q71

$$u = e^z + z \Rightarrow \frac{du}{dz} = e^z + 1 \Rightarrow dz = \frac{du}{e^z+1}$$

$$\Rightarrow \int \frac{e^2+1}{e^2+z}\, dz = \int \frac{1}{u}\, du = \ln|u| + c$$

$$= \ln|e^z + z| + c \qquad \Rightarrow \int_0^1 \frac{e^z+1}{e^z+z}\, dz = \ln(e+1).$$

Q87

$$u = 2x \Rightarrow \frac{du}{dx} = 2 \Rightarrow dx = \frac{du}{2} \Rightarrow$$

$$\int_0^2 f(2x)\, dx = \frac{1}{2}\int_{2\cdot 0}^{2\cdot 2} f(u)\, du = \frac{1}{2}\int_0^4 f(u)\, du = \frac{10}{2} = 5.$$