Derivatives in Nature

The derivative is a fundamental concept throughout all nature.

Examples

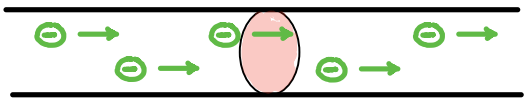
Motion :

$s(t)$ = position at time t

$\frac{ds}{dt} = v(t)$ = velocity at time t

$\frac{d^2s}{dt^2} = \frac{dv}{dt} = a(t)$ = acceleration at time t

Electrical Current :



$Q(t)$ = Total electrical charge (ie number of electrons) which has passed through red surface after time t.

$\frac{dQ}{dt}$ = Electrical current at time t.

Chemical Reactions :

$$aA + bB \longrightarrow cC$$

a molecules of form A + b molecules of form B gives c molecules of form C

$aA + bB$: Reactant Molecules

cC : Product Molecule

$R(t)$ = number of times reaction has occurred at time t in a one litre volume

generally in mole units

$6.022 \times 10^{23} = 1$ unit

$$\text{Rate of Reaction} = \frac{dR}{dt}$$

$[C](t)$ = concentration of C at time t

= number of C molecules in one litre volume at time t

$$[C](t) = c\,R(t) + [C](0)$$

$[C](0)$: concentration at start of reaction

$$\Rightarrow \frac{d[C]}{dt} = c\frac{dR}{dt} \Rightarrow \frac{dR}{dt} = \frac{1}{c}\frac{d[C]}{dt}$$

$$\Rightarrow$$

$$\text{Rate of Reaction} = \frac{1}{c}\frac{d[C]}{dt} \left(= \frac{-1}{a}\frac{d[A]}{dt} = \frac{-1}{b}\frac{d[B]}{dt} \right)$$

Population Growth :

$P(t)$ = Population size at time t

$B(t)$ = Total number of people ever born at time t

$D(t)$ = Total number of people who have ever died at t

$$P(t) = B(t) - D(t)$$

$$\frac{dP}{dt} = \frac{dB}{dt} - \frac{dD}{dt}$$

$\frac{dP}{dt}$: Growth rate

$\frac{dB}{dt}$: Birth rate

$\frac{dD}{dt}$: Death rate

Economic Growth Rate :

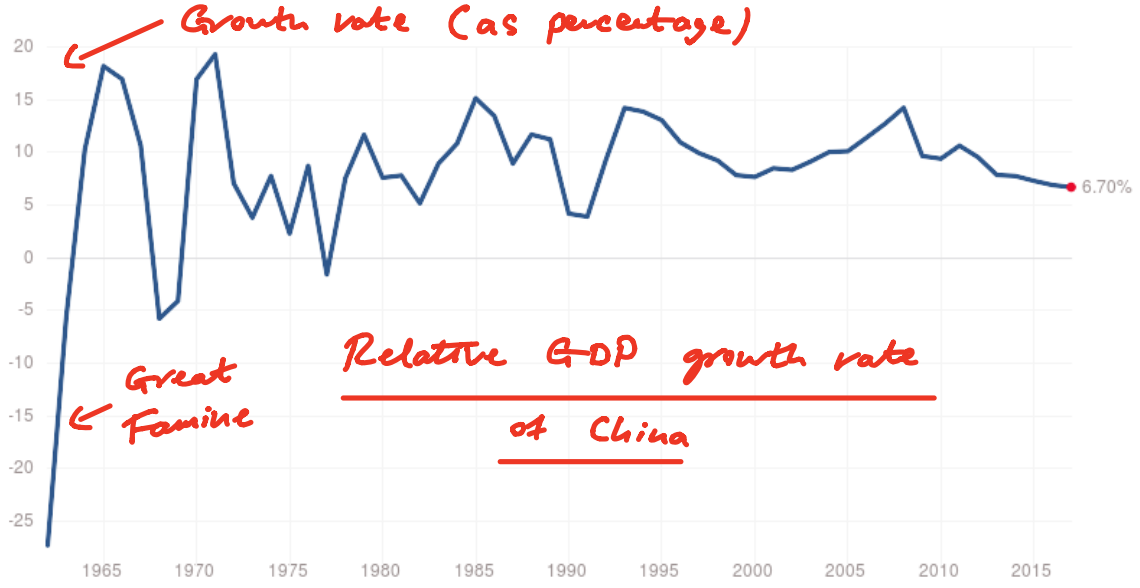
(GDP)

$g(t)$ = Gross Domestic Product at time t

← total value of all goods and services in a country

$\frac{g'(t)}{g(t)}$ = Relative growth rate of GDP at time t

When someone says China has 6% GDP growth rate that means $\frac{g'(t)}{g(t)} = 0.06$



Relative GDP growth rate of China