# MATH 1A MIDTERM 1 (PRACTICE 2)

## PROFESSOR PAULIN

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| **DO NOT TURN OVER UNTIL INSTRUCTED TO DO SO.** |
| **CALCULATORS ARE NOT PERMITTED** |
| **YOU MAY USE YOUR OWN BLANK PAPER FOR ROUGH WORK** |
| **SO AS NOT TO DISTURB OTHER STUDENTS, EVERYONE MUST STAY UNTIL THE EXAM IS COMPLETE** |
| **REMEMBER THIS EXAM IS GRADED BY A HUMAN BEING. WRITE YOUR SOLUTIONS NEATLY AND COHERENTLY, OR THEY RISK NOT RECEIVING FULL CREDIT** |
| **THIS EXAM WILL BE ELECTRONICALLY SCANNED. MAKE SURE YOU WRITE ALL SOLUTIONS IN THE SPACES PROVIDED. YOU MAY WRITE SOLUTIONS ON THE BLANK PAGE AT THE BACK BUT BE SURE TO CLEARLY LABEL THEM** |

Name and section: ______________________________

GSI's name: ______________________________

**This exam consists of 5 questions. Answer the questions in the spaces provided.**

1. Determine the domains of the following functions:

   (a) (15 points)

$$\ln(\frac{x+1}{x+4})$$

**Solution:**

$x+1>0$, $x+4>0$ $\Rightarrow$ $x>-1$, $x>-4$ $\Rightarrow$ $x>-1$

$x+1<0$, $x+4<0$ $\Rightarrow$ $x<-1$, $x<-4$ $\Rightarrow$ $x<-4$

$\Rightarrow$ Domain is $(-\infty,-4)\cup(-1,\infty)$

   (b) (10 points)

$$\arcsin(3-2x)$$

**Solution:**

Domain of $\arcsin(x)$ is $[-1,1]$

$-1\le 3-2x\le 1 \iff -4\le -2x\le -2 \iff 2\ge x\ge 1$

$\Rightarrow$ Domain is $[1,2]$

2. Let $f(x) = \begin{cases} \frac{x^2+kx-2}{x^2+2x-3} & \text{if } x \neq 1 \\ l & \text{if } x = 1 \end{cases}$ for some real number $k$ and $l$.

Determine what values of $k$ and $l$ make $f(x)$ continuous at $x = 1$? Carefully justify why $f(x)$ is continuous at $x = 1$ for these values.

**Solution:**

$$\lim_{x \to 1} x^2 + kx - 2 = 1^2 + k - 2 = k - 1$$

$$\lim_{x \to 1} x^2 + 2x - 3 = 1^2 + 2 \cdot 1 - 3 = 0$$

$$k - 1 \neq 0 \implies \lim_{x \to 1} \frac{x + kx - 2}{x^2 + 2x - 3} \quad \text{DNE}$$

$$k - 1 = 0 \implies k = 1$$

$$\lim_{x \to 1} \frac{x^2 + x - 2}{x^2 + 2x - 3} = \lim_{x \to 1} \frac{(x+2)(x-1)}{(x+3)(x-1)}$$

$$= \lim_{x \to 1} \frac{x+2}{x+3} = \frac{3}{4}$$

$\implies f$ continuous at $x = 1$ if $k = 1$

and $l = \frac{3}{4}$.

3. (25 points) Calculate (using the limit laws) the following limits. If a limit does not exist determine if it is $\infty$, $-\infty$ or neither.

(a)
$$\lim_{x\to 0}(e^{x^2+x} + x^2 + 1)$$

**Solution:**

$$\lim_{x\to 0} x^2+x = 0^2+0 = 0 \Rightarrow \lim_{x\to 0} e^{(x^2+x)} = e^0 = 1$$

$$\lim_{x\to 0} x^2+1 = 0^2+1 = 1$$

$$\Rightarrow \lim_{x\to 0} e^{(x^2+x)} + x^2+1 = 2$$

(b)
$$\lim_{x\to 1^+}\frac{x^2-9}{x^2+2x-3}$$

**Solution:**

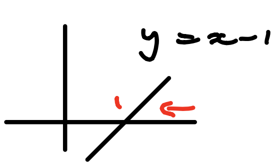
$x \neq -3$

$$\frac{x^2-9}{x^2+2x-3} = \frac{(x-3)(x+3)}{(x+3)(x-1)} = \frac{x-3}{x-1}$$

$$\lim_{x\to 1^+} x-3 = 1-3 = -2 < 0$$

$$\Rightarrow \lim_{x\to 1^+}\frac{x^2-9}{x^2+2x-3} = -\infty$$

$$\lim_{x\to 1^+} x-1 = 0^+$$



(c)

$$\lim_{x\to\infty} (\ln(3+x) - \ln(1+x))$$

**Solution:**

$$\ln(3+x) - \ln(1+x) = \ln\left(\frac{3+x}{1+x}\right)$$

$$\lim_{x\to\infty} \frac{3+x}{1+x} = \lim_{x\to\infty} \frac{\frac{3}{x}+1}{\frac{1}{x}+1} = \frac{1}{1} = 1$$

$$\Rightarrow \lim_{x\to\infty} (\ln(3+x) - \ln(1+x)) = \ln(1) = 0$$

(d)

$$\lim_{x\to 1} 2^{\arcsin\left(\frac{x^2-2x+1}{x-1}\right)}$$

**Solution:**

$x \neq 1$

$$\frac{x^2-2x+1}{x-1} = \frac{(x-1)(x-1)}{x-1} = x-1$$

$x-1$, $\arcsin(x)$, $2^x$ continuous $\Rightarrow$ $2^{\arcsin(x-1)}$ continuous

$1$ in domain of $2^{\arcsin(x-1)}$

$$\Rightarrow \lim_{x\to 1} 2^{\arcsin\left(\frac{x^2-2x+1}{x-1}\right)} = 2^{\arcsin(1-1)} = 2^0 = 1$$

4. (a) (20 points) Prove, using $\epsilon, \delta$ methods, that

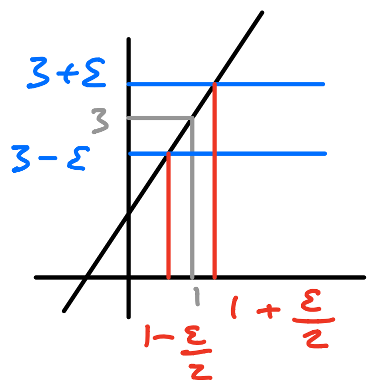
$$\lim_{x \to 1} \frac{2x^2 - x - 1}{x - 1} = 3.$$

**Solution:**

$$x \neq 1 \Rightarrow \frac{2x^2 - x - 1}{x-1} = \frac{(2x+1)(x-1)}{(x-1)} = 2x+1$$

Must prove $\lim_{x \to 1} 2x+1 = 3$

Let $\varepsilon > 0$




let $\delta = \frac{\varepsilon}{2}$

$0 < |x-1| < \frac{\varepsilon}{2}$

$\Rightarrow 2|x-1| < \varepsilon$

$\Rightarrow |2(x-1)| < \varepsilon$

$\Rightarrow |(2x+1) - 3| < \varepsilon$

$\Rightarrow \lim_{x \to 1} 2x+1 = 3$

$\Rightarrow \lim_{x \to 1} \frac{2x^2 - x - 1}{x-1} = 3$

(b) (5 points) Is $\frac{2x^2-x-1}{x-1}$ differentiable at $x = 1$?

**Solution:**

No, $f(1)$ is undefined

5. (a) (15 points) Using the direct definition of the derivative, calculate the derivative of the function

$$f(x) = x^{3/2}.$$

**Solution:**

Let $x > 0$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^{3/2} - x^{3/2}}{h} = \lim_{h \to 0} \frac{\left((x+h)^{3/2} - x^{3/2}\right)\left((x+h)^{3/2} + x^{3/2}\right)}{h\left((x+h)^{3/2} + x^{3/2}\right)}$$

$$= \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h\left((x+h)^{3/2} + x^{3/2}\right)} = \lim_{h \to 0} \frac{3x^2 + 3xh + h^2}{(x+h)^{3/2} + x^{3/2}}$$

$$= \frac{3x^2}{2x^{3/2}} = \frac{3}{2}\sqrt{x}$$

(b) (10 points) Does there exist a tangent line to $y = x^{3/2}$ which contain the point $(1, 0)$? Carefully justify your answer.

**Solution:**

Tangent line at $x = a$ is $y - a^{3/2} = \frac{3}{2}\sqrt{a}\,(x - a)$

$(1, 0)$ on tangent $\Rightarrow$ $-a^{3/2} = \frac{3}{2}\sqrt{a}\,(1 - a)$

$\Rightarrow$ $-a = \frac{3}{2}(1 - a)$

$\Rightarrow$ $\frac{1}{2}a = \frac{3}{2}$ $\Rightarrow$ $a = 3$

$\Rightarrow$ $(1, 0)$ is in tangent line at $x = 3$.

END OF EXAM