Maximum and Minimum Values

f - function with domain D

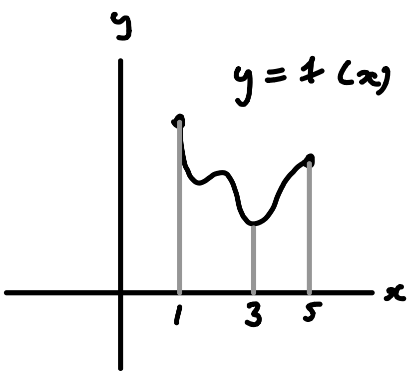
Definition

Given c in D, we say $f(c)$ is the

1/ **Absolute max** of f if $f(c) \geq f(x)$ for all x in D.

2/ **Absolute min** of f if $f(c) \leq f(x)$ for all x in D.

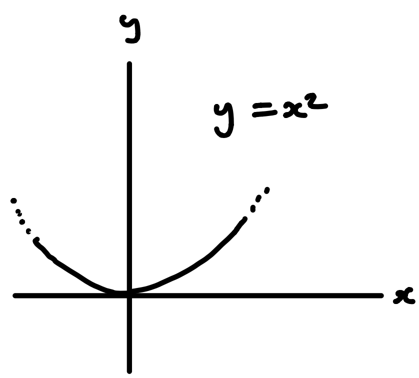
Examples



$D = [1,5]$

$f(1)$ absolute max

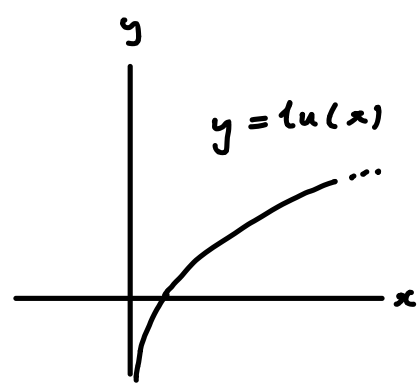
$f(3)$ absolute min



$D = \mathbb{R}$

$f(0) = 0^2 = 0$ abs. min

No absolute max



$D = (0,\infty)$

No abs. max. or abs. min.

Definition

We say $f(c)$ is a **local max** of f if

A/ $f(x)$ is defined for all x in some open interval I containing c.

and

B/ $f(c) \geq f(x)$ for all x in I

Definition

We say $f(c)$ is a local min of f if

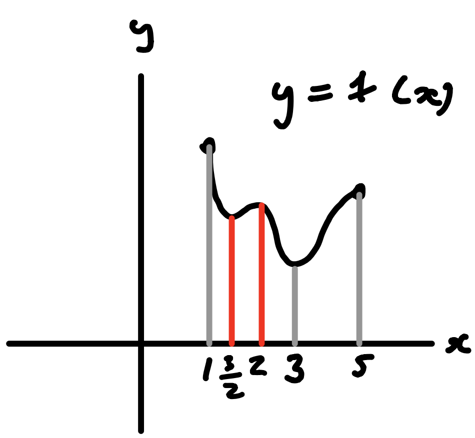
A/ $f(x)$ is defined for all x in some open interval I containing c.

and

B/ $f(c) \leq f(x)$ for all x in I

Warning: By convention, local max/min cannot occur at endpoints of a domain. Absolute max/min can occur at endpoints

Examples



$\Rightarrow$

$f(1)$ abs. max, not local max

$f(\frac{3}{2})$ local min only

$f(2)$ local max only

$f(3)$ abs. min and local min

$f(5)$ neither abs. max / local max

Definition

We say c is a <u>critical number</u> of f if

1/ c in domain D of f.

2/ $f'(c) = 0$ <u>or</u> $f'(c)$ DNE

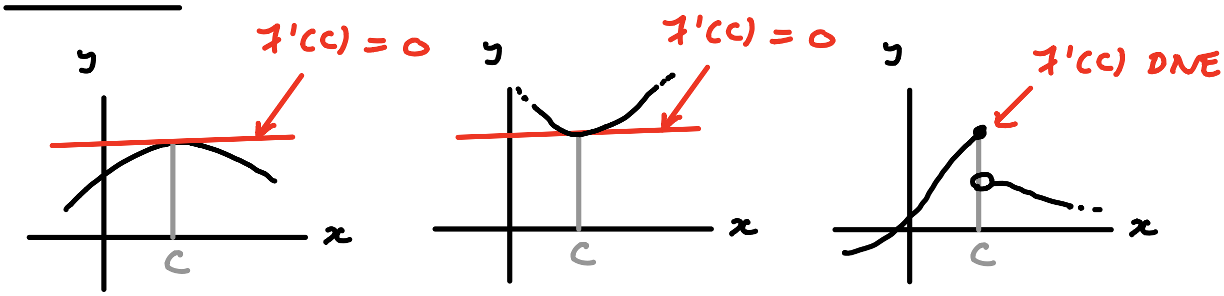
Includes endpoints of D if they exist

Important Fact :

converse is <u>not</u> true

$$\boxed{f(c) \text{ a local max/min} \Rightarrow c \text{ is a critical number of } f}$$

Examples :



Extreme Value Theorem

Closed Interval

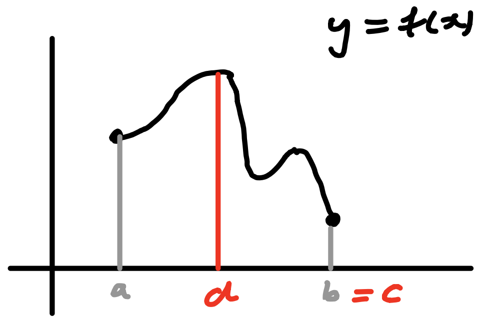
Let f be a continuous function with domain $[a,b]$.

Then there exists c, d in $[a,b]$ such that

$f(c)$ = absolute min of f

$f(d)$ = absolute max of f

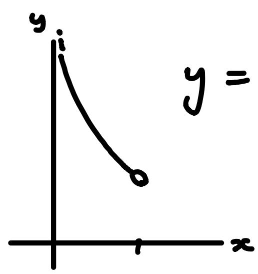
Example



Warning : If $D \neq [a,b]$ or f not continuous the conclusion is not valid.

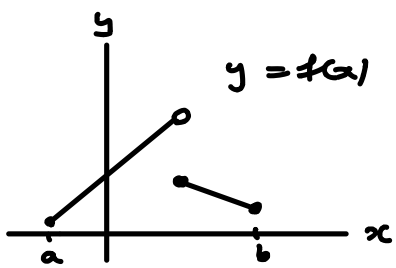
Examples

Open



$y = \frac{1}{x}$, $D = (0,1)$

$\frac{1}{x}$ has no abs. max in $(0,1)$



f has no abs. max in $[a,b]$

Observation :

$f(c)$ abs. max / min $\Rightarrow$ $f(c)$ local max / min or c an endpoint of D $\Rightarrow$ c a critical number of f

Finding Absolute max/min of continuous f on $[a,b]$:

1) Find all critical numbers of f in $[a,b]$ ← always include a,b

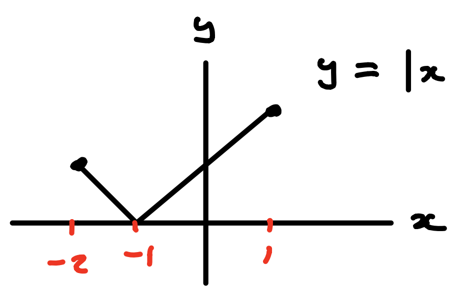
2) Evaluate $f(c)$ for all critical numbers

3) The largest is abs. max. The smallest is abs. min

Example $f(x) = \sqrt{x^2+2x+1}$ on $D = [-2,1]$

$$f(x) = \sqrt{x^2+2x+1} = \sqrt{(x+1)^2} = |x+1|$$

$$\Rightarrow f(x) = \begin{cases} x+1 & \text{if } x+1 \geq 0 \iff 1 \geq x \geq -1 \\ -x-1 & \text{if } x+1 < 0 \iff -2 \leq x < -1 \end{cases}$$



$\Rightarrow$ $-2, -1, 1$ are critical numbers

$f(-2) = 1$

$f(-1) = 0$ $\Rightarrow$ 2 abs. max, 0 abs. min

$f(1) = 2$