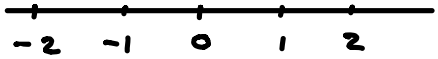
Functions

Calculus = Study of how different quantities vary with respect to each other.

$\mathbb{R} = (-\infty, \infty)$ = real numbers ← interval notation ← decimals

Number Line = $\mathbb{R}$



D a subset of $\mathbb{R}$ ← Often written $D \subset \mathbb{R}$

Examples

$(0, \infty)$ = all non-zero positive real numbers

$\mathbb{N} = \{1, 2, 3, 4, \ldots\}$ = natural numbers

$\mathbb{Z} = \{\ldots, -1, 0, 1, 2, \ldots\}$ = integers

$[1, 2) = x$ in $\mathbb{R}$ such that $1 \leq x < 2$

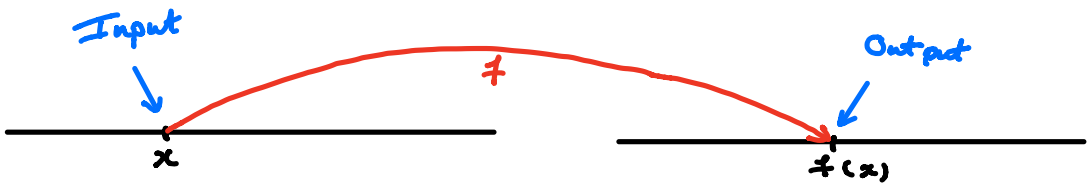
↑ Endpoint included ↑ Endpoint not included

Definition

A **function** f, with domain D, is a rule which assigns to every x in D **exactly one** real number, denoted $f(x)$.

The subset of all possible $f(x)$ is called the **range of f**

Crude Visualization:



Example

Motion in a straight line (on $\mathbb{R}$) between 12pm and 1pm

x = number of minutes after 12pm

$f(x)$ = position at time 12 : x pm

Domain = $[0, 60]$ (12pm, 1pm)

Four Key Ways to Represent a Function :

1/ Verbally ← Generally not so useful

2/ Numerically ← Using Table

3/ Visually ← Using a graph

4/ Algebraically ← By explicit formula

} Most useful

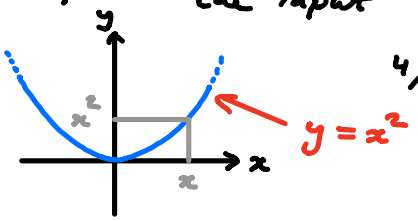
Example

1/ The function which squares the input

2/

x	$f(x)$
1	1
2	4
3	9

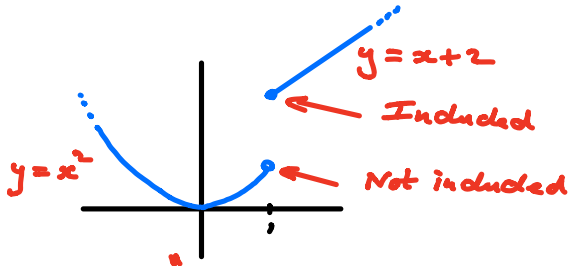
3/



4/ $f(x) = x^2$

Example (Piecewise defined)

$$f(x) = \begin{cases} x^2 & \text{if } x < 1 \\ x+2 & \text{if } x \geq 1 \end{cases}$$



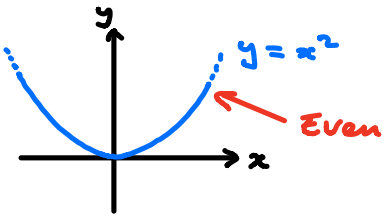
Elementary Properties ("if and only if")

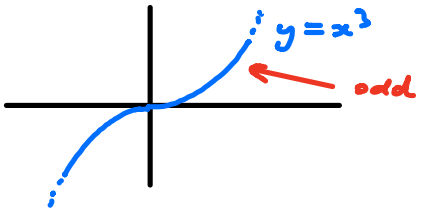
- f **even** $\iff f(-x) = f(x)$ for all x in domain
- f **odd** $\iff f(-x) = -f(x)$ for all x in domain

Fact :

f even $\iff$ $y = f(x)$ same after reflecting in y-axis

f odd $\iff$ $y = f(x)$ same after rotating by π (radians) about $(0, 0)$





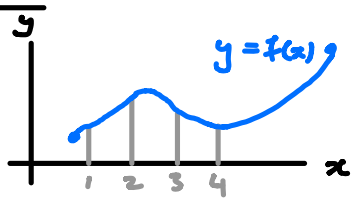
We say f <u>increasing</u> on interval I if

$x_1 < x_2$ in I $\Rightarrow$ ("implies") $f(x_1) < f(x_2)$

We say f <u>decreasing</u> on interval I if

$x_1 < x_2$ in I $\Rightarrow$ ("implies") $f(x_1) > f(x_2)$

Example



f <u>increasing</u> on $[1,2]$

$\Rightarrow$ f <u>decreasing</u> on $[3,4]$