

MATH 1A FINAL (PRACTICE 3)
PROFESSOR PAULIN

**DO NOT TURN OVER UNTIL
INSTRUCTED TO DO SO.**

CALCULATORS ARE NOT PERMITTED

**YOU MAY USE YOUR OWN BLANK
PAPER FOR ROUGH WORK**

**SO AS NOT TO DISTURB OTHER
STUDENTS, EVERYONE MUST STAY
UNTIL THE EXAM IS COMPLETE**

**REMEMBER THIS EXAM IS GRADED BY
A HUMAN BEING. WRITE YOUR
SOLUTIONS NEATLY AND
COHERENTLY, OR THEY RISK NOT
RECEIVING FULL CREDIT**

**THIS EXAM WILL BE ELECTRONICALLY
SCANNED. MAKE SURE YOU WRITE ALL
SOLUTIONS IN THE SPACES PROVIDED.
YOU MAY WRITE SOLUTIONS ON THE
BLANK PAGE AT THE BACK BUT BE
SURE TO CLEARLY LABEL THEM**

Name: _____

Student ID: _____

GSI's name: _____

This exam consists of 10 questions. Answer the questions in the spaces provided.

1. Calculate the following. You do not need to simplify your answers.

(a) (10 points)

$$\frac{d}{dx} \frac{\ln(x^2 + 2)}{\tan(x)}$$

Solution:

(b) (15 points)

$$\frac{d}{dx} \cos(x^{e^x})$$

Solution:

2. Calculate the following (you do not need to use the (ϵ, δ) -definition):

(a) (10 points)

$$\lim_{x \rightarrow 0} \frac{\tan(2x)}{x}$$

Solution:

(b) (15 points)

$$\lim_{x \rightarrow -\infty} \sqrt{(9x^2 - x)} + 3x$$

Hint: Rationalize.

Solution:

3. Calculate the following (you do not need to use the Riemann sum definition):

(a) (10 points)

$$\int x^3 \sqrt{x^2 + 1} dx$$

Solution:

(b) (10 points)

$$\int_1^2 \frac{e^{1/x^2}}{x^3} dx$$

Solution:

4. (25 points) The ration of carbon-14 to carbon-12 in an ancient wooden artifact is 60 percent that of living organic matter. How old is the artifact?

Hint: The half-life of carbon-14 is 5730 years.

Solution:

5. (25 points) Sketch the following curve. Be sure to indicate asymptotes, local maxima and minima and concavity. Show your working on this page and draw the graph on the next page.

$$y = \frac{x^3}{1 + x^2}$$

Solution:

Solution (continued) :

6. (25 points) Show that the tangent line to the curve

$$y = \int_{-3x}^{x^2} te^{t^2} dt$$

at $x = 3$, does not contain $(0, 0)$.

Solution:

7. (25 points) Find the area of the largest rectangle that can be inscribed in an equilateral triangle with sides of length 2 if one side of the rectangle lies on the base of the triangle.

Solution:

8. Two cars are travelling directly towards each other on a straight road. The first car is travelling at 3 metres per second. The second car is travelling at 6 metres per second. When they are 6 metres apart they simultaneously apply the brakes. The first car decelerates at a constant rate of 2 metres per second per second. The second car decelerates at a constant rate of 4 metres per second per second.

- (a) (15 points) How long after applying the brakes will the cars collide? Carefully justify your answer.

Solution:

- (b) (5 points) How far will the second car travel between applying the brakes and colliding with the first car?

Solution:

9. (25 points) Calculate the following limit:

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{n+2i}{n^2+in}$$

Solution:

10. (25 points) Calculate the volume of a pyramid with a square 2 by 2 base and height 1.

Solution: