MATH 1A FINAL (PRACTICE 2)
PROFESSOR PAULIN

DO NOT TURN OVER UNTIL INSTRUCTED TO DO SO.

CALCULATORS ARE NOT PERMITTED

YOU MAY USE YOUR OWN BLANK PAPER FOR ROUGH WORK

SO AS NOT TO DISTURB OTHER STUDENTS, EVERYONE MUST STAY UNTIL THE EXAM IS COMPLETE

REMEMBER THIS EXAM IS GRADED BY A HUMAN BEING. WRITE YOUR SOLUTIONS NEATLY AND COHERENTLY, OR THEY RISK NOT RECEIVING FULL CREDIT

THIS EXAM WILL BE ELECTRONICALLY SCANNED. MAKE SURE YOU WRITE ALL SOLUTIONS IN THE SPACES PROVIDED. YOU MAY WRITE SOLUTIONS ON THE BLANK PAGE AT THE BACK BUT BE SURE TO CLEARLY LABEL THEM

Name: _____

Student ID: _____

GSI's name: _____

This exam consists of 10 questions. Answer the questions in the spaces provided.

1. Calculate the following. You do not need to simplify your answers.

   (a) (10 points)

   $$\frac{d}{dx}\frac{\tan(\sin(e^x))}{x}$$

   Solution:

   (b) (15 points)

   $$\frac{d}{dx}\cos(x)^{\sin(\sqrt{x})}$$

   Solution:

2. Calculate the following (you do not need to use the (ϵ, δ)-definition):

(a) (10 points)

$$\lim_{x \to 0} \frac{\cos(2x^2) - 1}{4x}$$

Solution:

(b) (15 points)

$$\lim_{x \to \infty} (1 + \frac{4}{x})^x$$

Hint: Rationalize.

Solution:

3. Calculate the following (you do not need to use the Riemann sum definition):

(a) (10 points)

$$\int \frac{\arccos(x)}{\sqrt{1-x^2}}dx$$

Solution:

(b) (10 points)

$$\int_{1/\pi}^{2/\pi} \frac{\sin(1/x)}{3x^2}dx$$

Solution:

4. (25 points) Calculate the equation of the tangent line at $x = 3$ of the following curve:

$$\frac{2y^3(x-3)}{x\sqrt{y}} + 1 = \frac{3}{x\sqrt{y}}.$$

Solution:

5. (25 points) Sketch the following curve. Be sure to indicate asymptotes, local maxima and minima and concavity. Show your working on this page and draw the graph on the next page.

$$y = \ln(\frac{x}{2x^2 - x^3})$$

Solution:

Solution (continued) :

6. (25 points) Show that the following equation has at least one real solution. Be sure to carefully justify you answer clearly stating any results you use from lectures.

$$x \arctan |x| + c = 0, \text{ where } c \text{ is a constant.}$$

Hint: Consider behavior at ∞.

Solution:

7. (25 points) A drink will be packaged in a cylindrical can with volume 40in^3. The top and bottom of the can cost 4 cents per square inch. The sides cost 3 cents per square inch. Determine the dimensions of the can that minimize the cost.

   Solution:

8. Two rockets are fired vertically into the air from the ground. The second rocket is launched four seconds after the first. The velocity of the first rocket is $v_1(t) = 6 - t$ metres per second and the velocity of the second is $v_2(t) = 10 - t$ metres per second, where t is the time in seconds after the first launch.

   (a) (15 points) How long after first the launch will both rockets be at the same height? What will this height be?

   Solution:

   (b) (10 points) Determine the total distance traveled by the first rocket at this time.

   Solution:

9. (25 points) Calculate the following limit:

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{n}{n^2 + 3i^2}$$

Hint: Remember that $\frac{A}{B} = \frac{1}{B/A}$.

Solution:

10. (25 points) Calculate the volume of the solid of revolution formed by rotating the region enclosed by $x = y^2$ (with $y \geq 0$), $x = -y$ and $x = y + 2$ around the line $y = 2$.

Solution:

END OF EXAM