Essential Functions

Polynomials

$f(x) = a_0 + a_1 x + \dots + a_n x^n$

If $a_n \neq 0$ say f has degree n

n = non-negative integer

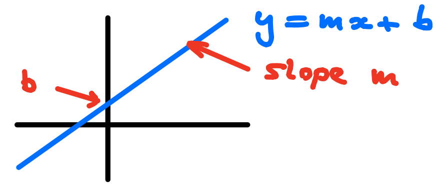
$a_0, a_1, \dots, a_n$ = constant real numbers

Called coefficients

Degree $\leq 1 \Rightarrow f$ <u>Linear</u> $\Rightarrow y = f(x)$ straight line

Domain of polynomial $= \mathbb{R}$

Range depends on degree



Power Functions

$f(x) = x^a$ (a fixed real number)

n = non-zero positive integer

$a = n \Rightarrow f$ polynomial

$a = 1/n$ (n positive integer) $\Rightarrow f(x) = x^{1/n} = \sqrt[n]{x}$

n^{th} root of x

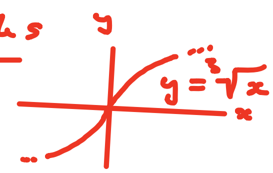
Conventions :

1/ $\sqrt[2]{x} = \sqrt{x}$

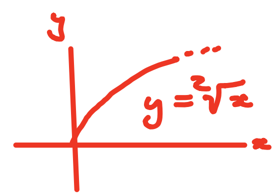
$x \geq 0$

2/ n even $\Rightarrow \sqrt[n]{x} \geq 0$

Examples

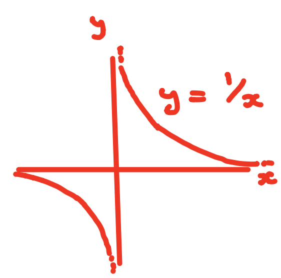


n <u>odd</u> $\Rightarrow$ Domain = Range = $\mathbb{R}$



n <u>even</u> $\Rightarrow$ Domain = Range = $[0, \infty)$

$f(x) = x^{-1} = \frac{1}{x}$ = reciprocal function



Rational Functions

$f(x) = \frac{P(x)}{Q(x)}$, P, Q polynomials

Range much harder to determine

Domain = x such that $Q(x) \neq 0$

Example $f(x) = \frac{x^2+2x+4}{x^2-1} \Rightarrow$ Domain = all $x \neq \pm 1$

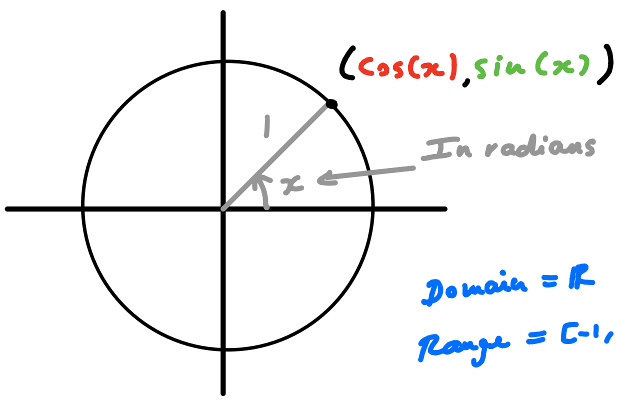
Algebraic Functions

f = function constructed using $+, -, \times, \div, \sqrt[n]{\ }$ starting from polynomials

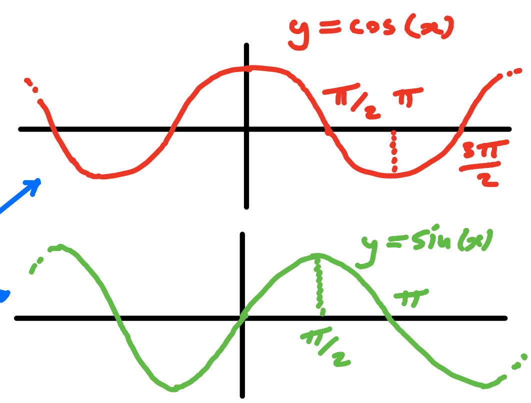
Domain = All x for which $f(x)$ makes sense

Example $f(x) = \frac{\sqrt[3]{x^2+1}}{x^2+1} - x\sqrt{x+1} \Rightarrow$ Domain = $[-1, \infty)$

Trigonometric Functions



$\Rightarrow$



Domain = $\mathbb{R}$

Range = $[-1, 1]$

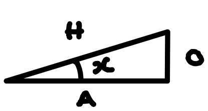
n an integer

$\sin(x) = 0 \Leftrightarrow x = n\pi$

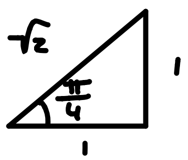
$\cos(x) = 0 \Leftrightarrow x = \frac{\pi}{2} + n\pi$

$\tan(x) = \frac{\sin(x)}{\cos(x)} \Rightarrow$ Domain = all $x \neq \frac{\pi}{2} + n\pi$

Range = $\mathbb{R}$



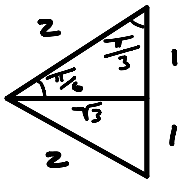
$\Rightarrow \sin(x) = \frac{O}{H}$, $\cos(x) = \frac{A}{H}$, $\tan(x) = \frac{O}{A}$



$$\Rightarrow \quad \sin\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$\tan(\pi/4) = 1$$

Useful to know



$$\Rightarrow \quad \sin(\pi/6) = \cos(\pi/3) = 1/2$$

$$\sin(\pi/3) = \cos(\pi/6) = \frac{\sqrt{3}}{2}$$

$$\tan(\pi/3) = \sqrt{3} \quad , \quad \tan(\pi/6) = \frac{1}{\sqrt{3}}$$