Homework 8

§10.4

Q3 a) $y = 7(t) \Rightarrow \frac{dy}{dt} = 0.05y + 3600$

b) $\frac{dy}{dt} = 0.05y + 3600 \Rightarrow \frac{dy}{dt} + (-0.05)y = 3600$

$\Rightarrow a(t) = -0.05 \;,\; b(t) = 3600 \;,\; A(t) = -0.05t$

$\Rightarrow y(t) = \frac{1}{e^{-0.05t}} \int e^{-0.05t} \cdot 3600 \, dt$

$= e^{0.05t} \left(\frac{3600}{-0.05} e^{-0.05t} + C \right)$

$= \frac{3600}{-0.05} + C e^{0.05t}$

$y(0) = 0 \Rightarrow \frac{3600}{-0.05} + C = 0 \Rightarrow C = \frac{3600}{0.05}$

$\Rightarrow y(t) = \frac{3600}{0.05} \left(e^{0.05t} - 1 \right)$

$\Rightarrow y(25) = \frac{3600}{0.05} \left(e^{1.25} - 1 \right)$

Q4

a) $y = f(t) \Rightarrow \frac{dy}{dt} = 0.06y + A$

b) $\Rightarrow a(t) = -0.06 \,,\; b(t) = A \,,\; A(t) = -0.06t$

$$\Rightarrow y(t) = \frac{1}{e^{-0.06t}} \int e^{-0.06t} \cdot A \, dt$$

$$= \frac{A}{-0.06} + C e^{0.06t}$$

$y(0) = 10000 \Rightarrow \frac{A}{-0.06} + C = 10000 \Rightarrow C = 10000 + \frac{A}{0.06}$

$$\Rightarrow y(t) = \frac{A}{-0.06} + \left(10000 + \frac{A}{0.06}\right) e^{0.06t}$$

c) $y(5) = 20000 \Rightarrow 20000 = \frac{A}{-0.06} + \left(10000 + \frac{A}{0.06}\right) e^{0.3}$

$$\Rightarrow A = \frac{20000 - 10000 e^{0.3}}{\frac{1}{-0.06} + \frac{e^{0.3}}{0.06}}$$

Q5

$y_1(t)$ = amount in Kelly's account

$$\Rightarrow \frac{dy_1}{dt} = 0.05\,y_1 + 1200 \quad \Rightarrow \quad a(t) = -0.05,\ b(t) = 1200$$

$$A(t) = -0.05t$$

$$\Rightarrow y_1(t) = \frac{1}{e^{-0.05t}} \int e^{-0.05t} \cdot 1200\, dt$$

$$= \frac{1200}{-0.05} + C e^{0.05t}$$

$$y_1(0) = 0 \Rightarrow C = \frac{1200}{+0.05} \Rightarrow y_1(t) = \frac{1200}{0.05}\left(-1 + e^{0.05t}\right)$$

$$y_1(20) = \frac{1200}{0.05}(-1 + e) \approx \$41,214$$

$y_2(t)$ = amount in John's account

$$\Rightarrow \frac{dy_2}{dt} = 0.05\,y_2 + 2400 \quad \Rightarrow \quad a(t) = -0.05,\ b(t) = 2400 \quad A(t) = -0.05t$$

$$\Rightarrow y_2(t) = \frac{1}{e^{-0.05t}} \int e^{-0.05t} \cdot 2400\, dt$$

$$= \frac{2400}{-0.05} + C e^{0.05t}$$

$y_2(0) = 0 \Rightarrow C = \frac{2400}{0.05} \Rightarrow y_2(t) = \frac{2400}{0.05}(-1 + e^{0.05t})$

$y_2(10) = \frac{2400}{0.05}(-1 + e^{0.5}) \approx \$31,139$

Kelly will have more.

Q7 / $y(t)$ = amount owed at time t

$\Rightarrow \frac{dy}{dt} = 0.075y - K$ (K: constant repayment rate)

$\Rightarrow a(t) = -0.075$

$b(t) = -K \Rightarrow A(t) = -0.075t$

$\Rightarrow y(t) = \frac{1}{e^{-0.075t}} \cdot \int e^{-0.075t} \cdot (-K)\, dt$

$= \frac{K}{0.075} + C \cdot e^{0.075t}$

$y(0) = 100000 \Rightarrow \frac{K}{0.075} + C = 100000$

$\Rightarrow C = 100000 - \frac{K}{0.075}$

$$\Rightarrow y(t) = \frac{K}{0.075} + \left(100000 - \frac{K}{0.075}\right) e^{0.075t}$$

$$y(10) = 0 \Rightarrow \frac{K}{0.075} + \left(100000 - \frac{K}{0.075}\right) e^{0.75} = 0$$

$$\Rightarrow K = \frac{-100000\, e^{0.75}}{\frac{1}{0.075} - \frac{e^{0.75}}{0.075}} \approx \$14,214 \text{ per year}$$

Q17 $y(t)$ = Culture size at time t $\Rightarrow \frac{dy}{dt} = 0.45y + e^{0.03t} + 2$

Q21 a) $y = f(t) \Rightarrow \frac{dy}{dt} = 0.04y - 2000 - 500t$

b) $\Rightarrow a(t) = -0.04$, $b(t) = -2000 - 500t$, $A(t) = -0.04t$

$$\Rightarrow y(t) = \frac{1}{e^{-0.04t}} \cdot \int e^{-0.04t} (-2000 - 500t)\, dt$$

$$= \frac{1}{e^{-0.04t}} \left(\int -2000\, e^{-0.04t}\, dt - 500 \int t\, e^{-0.04t}\, dt \right)$$

$f(t) = t$, $g(t) = e^{-0.04t}$

$f'(t) = 1$, $G(t) = \frac{1}{-0.04} e^{-0.04t}$

$$\Rightarrow \int t e^{-0.04t}\, dt = \frac{t}{-0.04} e^{-0.04t} - \frac{1}{(0.04)^2} e^{-0.04t} + \text{Constt}$$

$$\Rightarrow y(t) = \frac{-2000}{-0.04} - 500\left(\frac{t}{-0.04} - \frac{1}{(0.04)^2}\right) + \frac{C}{e^{-0.04t}}$$

$$y(0) = 100{,}000 \Rightarrow \frac{-2000}{-0.04} + \frac{500}{(0.04)^2} + C = 100000$$

$$\Rightarrow C = 100000 - \frac{2000}{0.04} - \frac{500}{(0.04)^2}$$

$$\Rightarrow y(t) = \frac{2000}{0.04} - 500\left(\frac{t}{-0.04} - \frac{1}{(0.04)^2}\right) + \left(100000 - \frac{2000}{0.04} - \frac{500}{(0.04)^2}\right) e^{0.04t}$$

c) Check the textbook.

Q23

a) $3000 - 500t = 0 \Rightarrow t = 6$ years

b) $\frac{dP}{dt} = 0.04P + 3000 - 500t$

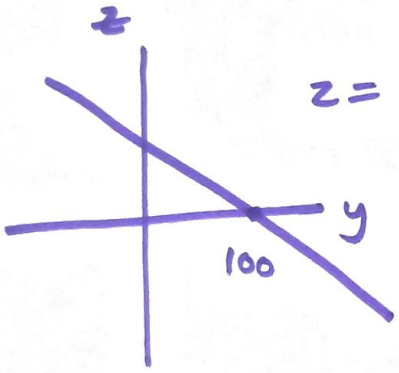
$P(0) = 10000$

§10.6

Q7

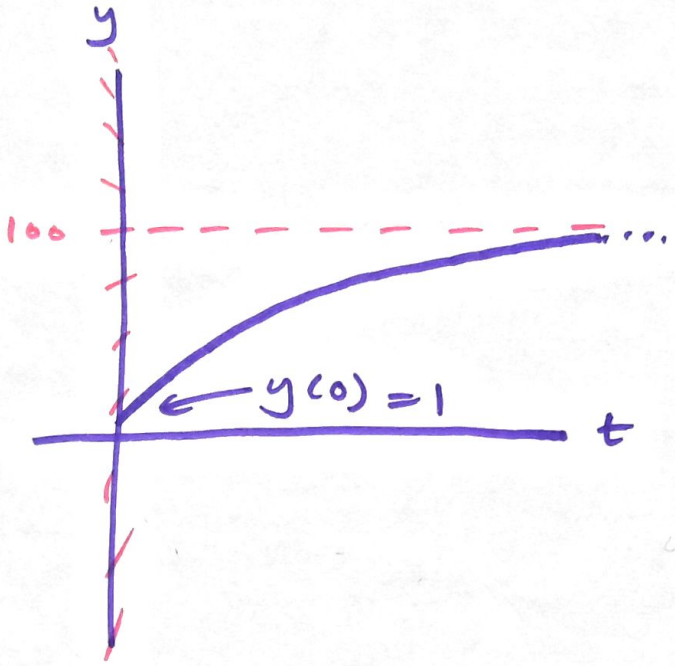
$$\frac{dy}{dt} = k(100 - y)$$

(k: positive constant; 100: percentage who don't have info)



$z = k(100 - y)$

$\Rightarrow$



Q8

$y(t)$ = distance dropped at time t

Conjecture: $\frac{dy}{dt} = ky$ (k: positive constant)

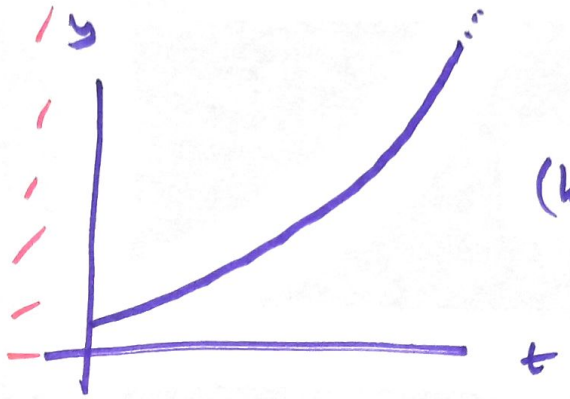
$\Rightarrow y(t) = Ae^{kt}$

For A a constant.

$y(0) = 0 \Rightarrow A = 0 \Rightarrow y(t) = 0$ ← This would mean the body can never move. This is clearly wrong.

Q12 $y(t)$ = population at time t

$$\Rightarrow \frac{dy}{dt} = \underbrace{k}_{\text{constant}} y^2$$

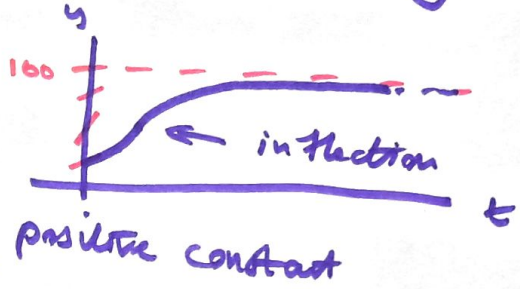


$(k > 0)$

Q14

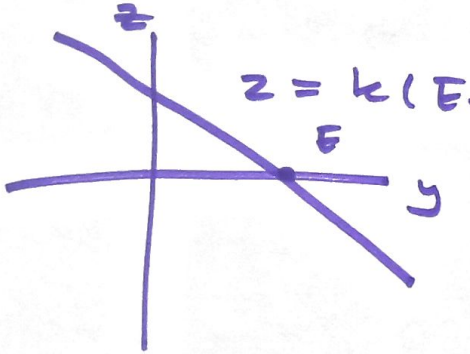
y = percentage advocating war $\Rightarrow$ $100 - y$ = percentage <u>not</u> advocating war

$$\Rightarrow \frac{dy}{dt} = \underbrace{k}_{\text{constant}} y(100 - y)$$

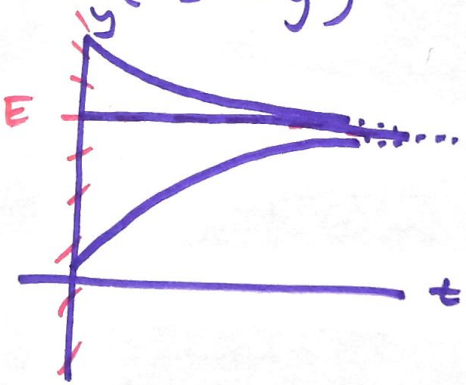


positive constant

Q15 $y = f(t) \Rightarrow \frac{dy}{dt} = \underbrace{k}_{\text{constant}}(E - y)$



$\Rightarrow$

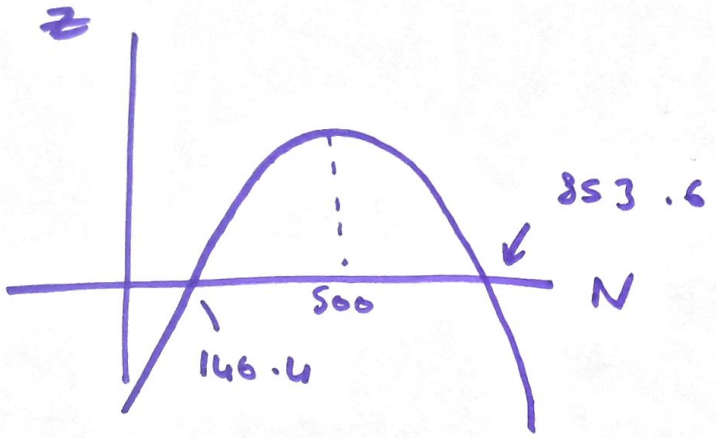


Q17

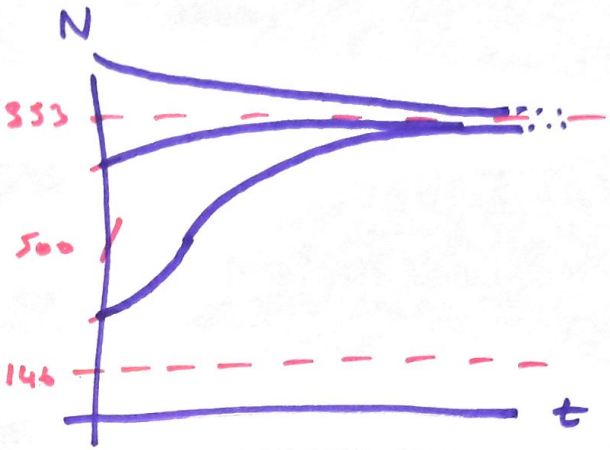
a) $\frac{dN}{dt} = \frac{0.4}{1000} N(1000 - N) - 50$

$$z = \frac{0.4}{1000} N(1000 - N) - 50$$

$$\Rightarrow$$



$\Rightarrow$



c) It is sustainable. It will not ever reach the carrying capacity.

Q20 $y(t)$ = account balance at time t $\Rightarrow$

$$\frac{dy}{dt} = 0.05y + P \Rightarrow a(t) = -0.05, \quad b(t) = P \Rightarrow A(t) = -0.05t$$

$$y(0) = 0$$

$$\Rightarrow y(t) = \frac{1}{e^{-0.05t}} \cdot \int e^{-0.05t} \cdot P \, dt$$

$$= \frac{P}{-0.05} + C e^{0.05t}$$

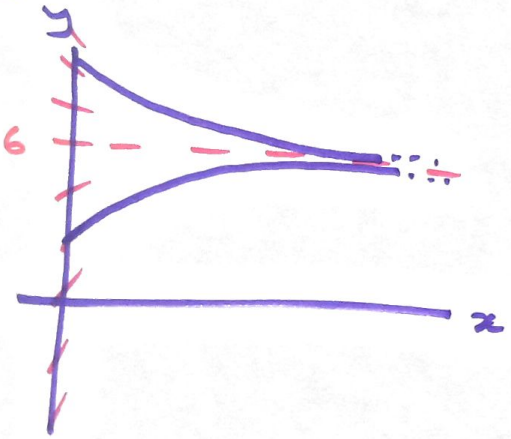
$y(0) = 0 \Rightarrow C = \frac{P}{0.05} \Rightarrow y(t) = \frac{P}{0.05}(-1 + e^{0.05t})$

$y(4) = 50000 \Rightarrow 50000 = \frac{P}{0.05}(-1 + e^{0.2})$

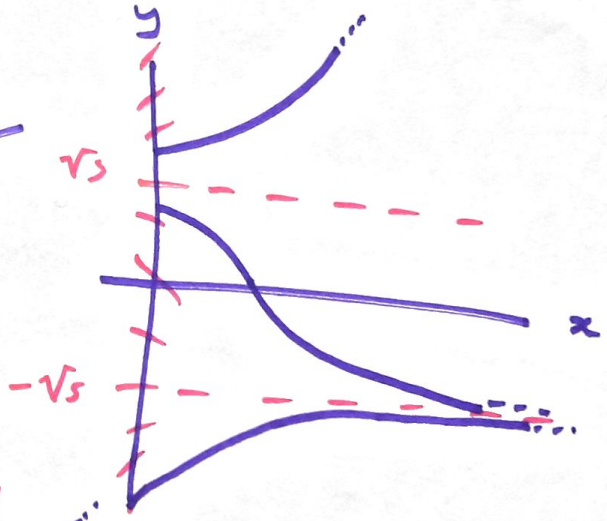
$\Rightarrow P = \frac{50000 \times (0.05)}{-1 + e^{0.2}}$ $ per year.

10.5

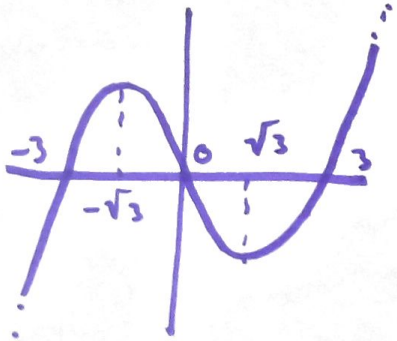
Q7



Q9



Q13



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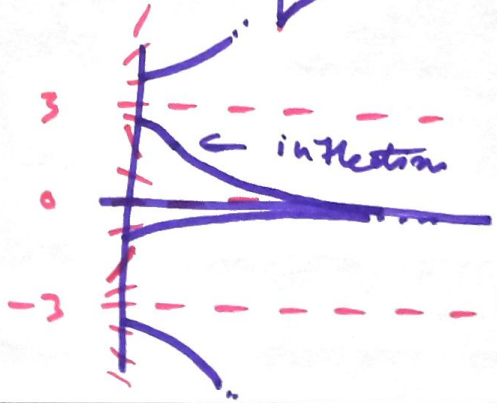
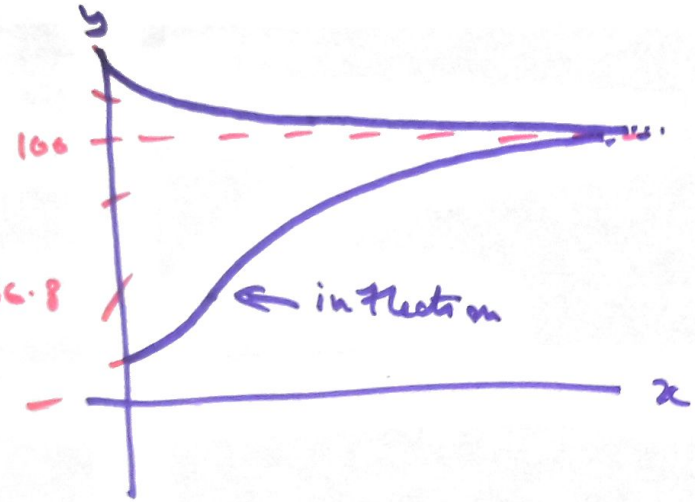
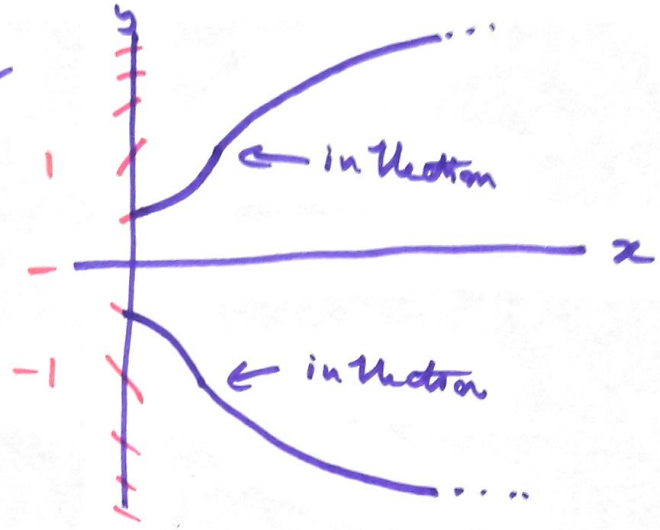


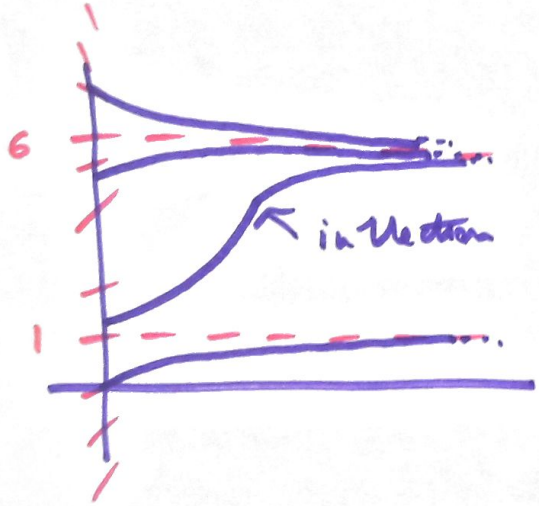
Q15



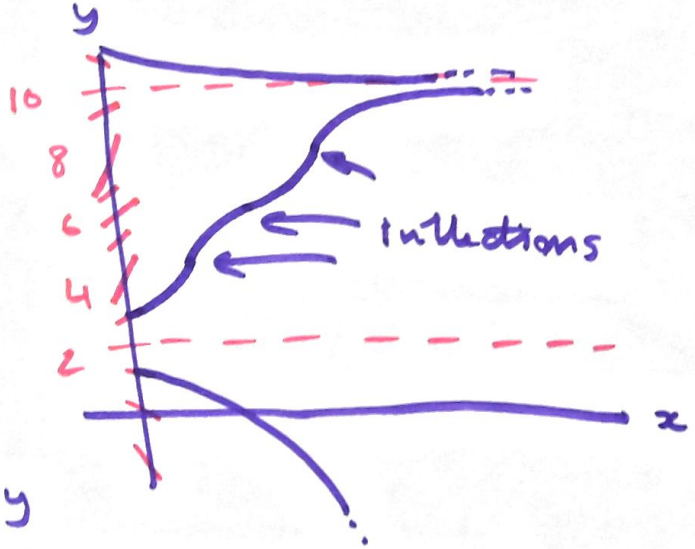
Q17



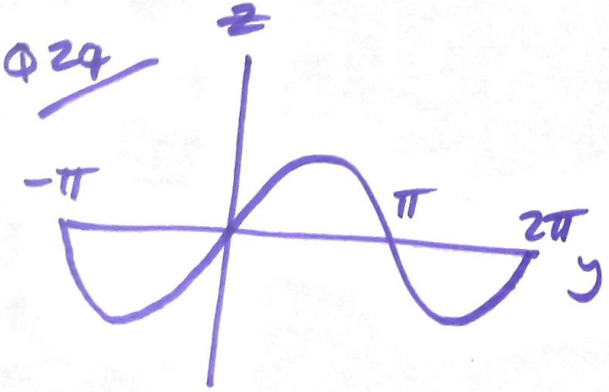
Q19



Q20



Q24



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