Homework 6

§4.5

Q1

$f(t) = 35000$, $r = 0.07$, $T = 5$ $\Rightarrow$ Present Value over $[0,5]$ $= \int_0^5 35000\, e^{-0.07t}\, dt$

$$= \frac{35000}{-0.07} e^{-0.07t} \Big|_0^5$$

$$= \frac{35000}{0.07} - \frac{35000}{0.07} e^{-0.35}.$$

Q2

$f(t) = 60000$, $r = 0.06$ $\Rightarrow$ Present Value over $[2,6]$ $= \int_2^6 60000\, e^{-0.06t}\, dt$

$$= \frac{60000}{-0.06} e^{-0.06t} \Big|_2^6 = \frac{60000}{-0.06} e^{-0.36} + \frac{60000}{0.06} e^{-0.12}$$

Q4

$f(t) = 25e^{-0.02t}$

$r = 0.08$

$T = 4$

$\Rightarrow$ Present Value over $[0,4]$ $= \int_0^4 25e^{-0.02t} \cdot e^{-0.08t}\,dt$

$= \int_0^4 25e^{-0.1t}\,dt = \frac{25}{-0.1} e^{-0.1t} \Big|_0^4$

$= \frac{25}{0.1} - \frac{25}{0.1} e^{-0.4}$

Q7

a) $f(t) = 30 + 5t$

$r = 0.1$

$T = 2$

$\Rightarrow$ Present Value over $[0,2]$ $= \int_0^2 (30+5t)e^{-0.1t}\,dt$

b) ~~Let~~ $f(t) = t$, $g(t) = e^{-0.1t}$

$f'(t) = 1$, $G(t) = -10e^{-0.1t}$ $\Rightarrow \int te^{-0.1t}\,dt = -10te^{-0.1t} - 100e^{-0.1t} + c$

$= (-10t - 100)e^{-0.1t} + c$

$\Rightarrow$ Present Value $= \int_0^2 30e^{-0.1t} + 5\int_0^2 te^{-0.1t}\,dt$

$$= -300 e^{-0.1t} \Big|_0^2 + (-50t - 500) e^{-0.1t} \Big|_0^2$$

$$= -300 e^{-0.2} + 300 + -600 e^{-0.2} + 500$$

$$= 800 - 900 e^{-0.2}.$$

Q4

a) $$\int_0^5 120 e^{-0.65t} \cdot 2\pi t \, dt$$

b) $$= 240\pi \int_0^5 t e^{-0.65t} \, dt$$

$f(t) = t$ $\quad g(t) = e^{-0.65t}$

$f'(t) = 1$ $\quad G(x) = \dfrac{1}{-0.65} e^{-0.65t}$ $\Rightarrow \displaystyle\int t e^{-0.65t} \, dt = \frac{t}{-0.65} e^{-0.65t} - \frac{1}{(0.65)^2} e^{-0.65t} + c$

$$\Rightarrow 240\pi \int_0^5 t e^{-0.65t} \, dt = 240\pi \cdot \left(\frac{t}{-0.65} - \frac{1}{(0.65)^2} \right) e^{-0.65t} \Big|_0^5$$

$$= 240\pi\left(\frac{5}{-0.65} - \frac{1}{(0.65)^2}\right)e^{-3.25} + \frac{240\pi}{(0.65)^2}.$$

Q10

$$\int_3^5 120e^{-0.65t} \cdot 2\pi t\,dt = 240\pi\left(\frac{t}{-0.65} - \frac{1}{(0.65)^2}\right)e^{-0.65t}\Big|_3^5$$

$$= 240\pi\left(\frac{5}{-0.65} - \frac{1}{(0.65)^2}\right)e^{-3.25} + 240\pi\left(\frac{3}{0.65} + \frac{1}{(0.65)^2}\right)e^{-1.95}.$$

Q12

$$\int_1^{10} \frac{22\pi t}{(t^2+10)^2}\,dt \Rightarrow \text{Let } u = t^2+10 \Rightarrow \frac{du}{dt} = 2t$$

$$\Rightarrow dt = \frac{du}{2t}$$

$$\Rightarrow \int \frac{22\pi t}{(t^2+10)^2}\,dt = \int \frac{11\pi}{u^2}\,du = \frac{-11\pi}{u} + C = \frac{-11.\pi}{(t^2+10)} + C$$

$$\Rightarrow \int_1^{10} \frac{22\pi t}{(t^2+10)^2}\,dt = \frac{-11.\pi}{(t^2+10)}\Big|_1^{10} = \frac{-11.\pi}{110} + \frac{11\pi}{11}$$

tons of lava.

§4.6

Q4 $\lim_{b\to\infty} \frac{1}{b} + \frac{1}{3} = \frac{1}{3}$

Q7 $\lim_{b\to\infty} 2 - \frac{1}{\sqrt{b+1}} = 2$

Q9 $\lim_{b\to\infty} \frac{5}{b^2+3} = 0$

Q11 $\lim_{b\to\infty} e^{-b/2} + 5 = 5$

Q13 $\int 1/x^2 \, dx = \frac{-1}{x} + C \Rightarrow \int_2^t 1/x^2 \, dx = -1/x \Big|_2^t = \frac{1}{2} - 1/t$

$\int_2^\infty 1/x^2 \, dx =$ area under $y = 1/x^2$ over $[2,\infty)$ $= \lim_{t\to\infty} \frac{1}{2} - 1/t = 1/2$.

Q15 $\int e^{-x/2} \, dx = -2e^{-x/2} + C \Rightarrow \int_0^t e^{-x/2} \, dx = -2e^{-x/2} \Big|_0^t = 2 - 2e^{-t/2}$

$\Rightarrow \int_0^\infty e^{-x/2} \, dx =$ area under $y = e^{-x/2}$ over $[0,\infty)$ $= \lim_{t\to\infty} 2 - 2e^{-t/2} = 2$.

Q19/

$$\int (14x+18)^{-4/5}\,dx = \frac{5}{14}(14x+18)^{1/5} + c$$

$$\Rightarrow \int_1^t (14x+18)^{-4/5}\,dx = \frac{5}{14}(4x+18)^{1/5}\Big|_1^t$$

$$= \frac{5}{14}(4t+18)^{1/5} - \frac{5}{14}22^{1/5}$$

$$\Rightarrow \int_1^\infty (14x+18)^{-4/5}\,dx = \lim_{t\to\infty} \frac{5}{14}(4t+18)^{1/5} - \frac{5}{14}22^{1/5} = \infty$$

$\Rightarrow$ Divergent, so area is infinite.

Q24/

$$\int e^{-3x}\,dx = \frac{-1}{3}e^{-3x} + c \Rightarrow \int_0^t e^{-3x}\,dx = \frac{-1}{3}e^{-3x}\Big|_0^t = \frac{1}{3} - \frac{1}{3}e^{-3t}$$

$$\Rightarrow \int_0^\infty e^{-3x}\,dx = \lim_{t\to\infty} \frac{1}{3} - \frac{1}{3}e^{-3t} = \frac{1}{3}.$$

Q28/

$$\int e^{2-x}\,dx = -e^{2-x} + c \Rightarrow \int_2^t e^{2-x}\,dx = -e^{2-x}\Big|_2^t$$

$$= 1 - e^{2-t}$$

$$\Rightarrow \int_2^{\infty} e^{2-x}\,dx = \lim_{t\to\infty} 1 - e^{2-t} = 1$$

Q33 $\quad u = x^3 - 1 \Rightarrow \frac{du}{dx} = 3x^2 \Rightarrow dx = \frac{du}{3x^2} \Rightarrow$

$$\int \frac{x^2}{\sqrt{x^3-1}}\,dx = \frac{1}{3}\int \frac{1}{\sqrt{u}}\,du = \frac{2}{3}\sqrt{u} + C = \frac{2}{3}\sqrt{x^3-1} + C$$

$$\Rightarrow \int_3^t \frac{x^2}{\sqrt{x^3-1}}\,dx = \frac{2}{3}\sqrt{x^3-1}\,\Big|_3^t = \frac{2}{3}\sqrt{t^3-1} - \frac{2}{3}\sqrt{8}$$

$$\Rightarrow \int_3^{\infty} \frac{x^2}{\sqrt{x^3-1}}\,dx = \lim_{t\to\infty}\left(\frac{2}{3}\sqrt{t^3-1} - \frac{2}{3}\sqrt{8}\right) = \infty \Rightarrow \text{Divergent.}$$

Q34 $\quad u = \ln(x) \Rightarrow \frac{du}{dx} = \frac{1}{x} \Rightarrow dx = x\,du \Rightarrow$

$$\int \frac{1}{x\ln(x)}\,dx = \int \frac{1}{u}\,du = \ln|u| + C = \ln|\ln(x)| + C$$

$$\Rightarrow \int_2^t \frac{1}{x\ln(x)}\,dx = \ln|\ln(x)|\,\Big|_2^t = \ln|\ln(t)| - \ln(\ln(2))$$

$$\Rightarrow \int_2^{\infty} \frac{1}{x\ln(x)}\,dx = \lim_{t\to\infty} \ln|\ln(t)| - \ln|\ln(2)| = \infty$$

$\Rightarrow$ Divergent.

Q42

$$u = 4 - x \Rightarrow \frac{du}{dx} = -1 \Rightarrow dx = -du$$

$$\Rightarrow \int \frac{1}{\sqrt{4-x}}\,dx = -\int \frac{1}{\sqrt{u}}\,du = -2\sqrt{u} + c$$

$$= -2\sqrt{4-x} + c$$

$$\Rightarrow \int_t^0 \frac{1}{\sqrt{4-x}}\,dx = -2\sqrt{4-x}\,\Big|_t^0 = 2\sqrt{4-t} - 4$$

$$\Rightarrow \int_{-\infty}^0 \frac{1}{\sqrt{4-x}}\,dx = \lim_{t\to-\infty} 2\sqrt{4-t} - 4 = \infty$$

$\Rightarrow$ Divergent.

Q44 $u = e^{-x} + 2 \Rightarrow \frac{du}{dx} = -e^{-x} \Rightarrow dx = \frac{du}{-e^{-x}}$

$$\Rightarrow \int \frac{e^{-x}}{(e^{-x}+2)^2}\,dx = -\int \frac{1}{u^2}\,du = \frac{1}{u} + C = \frac{1}{e^{-x}+2} + C$$

$$\Rightarrow \int_0^t \frac{e^{-x}}{(e^{-x}+2)^2}\,dx = \left.\frac{1}{e^{-x}+2}\right|_0^t = \frac{1}{e^{-t}+2} - \frac{1}{3}$$

$$\Rightarrow \int_0^\infty \frac{e^{-x}}{(e^{-x}+2)^2}\,dx = \lim_{t\to\infty} \frac{1}{e^{-t}+2} - \frac{1}{3} = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$

$$\int_t^0 \frac{e^{-x}}{(e^{-x}+2)^2}\,dx = \left.\frac{1}{e^{-x}+2}\right|_t^0 = \frac{1}{3} - \frac{1}{e^{-t}+2}$$

$$\Rightarrow \int_{-\infty}^0 \frac{e^{-x}}{(e^{-x}+2)^2}\,dx = \lim_{t\to-\infty} \frac{1}{3} - \frac{1}{e^{-t}+2} = \frac{1}{3}$$

$$\Rightarrow \int_{-\infty}^\infty \frac{e^{-x}}{(e^{-x}+2)^2}\,dx = \frac{1}{6} + \frac{1}{3} = \frac{1}{2}.$$

Q47 $u = \ln(x) \Rightarrow \frac{du}{dx} = \frac{1}{x} \Rightarrow dx = x\,du$

$$\Rightarrow \int \frac{k}{x(\ln(x))^{k+1}}\,dx = \int \frac{k}{u^{k+1}}\,du = \frac{k}{-k}u^{-k} + C$$

$$= \frac{-1}{u^k} + C = \frac{-1}{(\ln(x))^k} + C$$

$$\Rightarrow \int_e^t \frac{k}{x(\ln(x))^{k+1}}\,dx = \left.\frac{-1}{(\ln(x))^k}\right|_e^t = 1 - \frac{1}{(\ln(t))^k}$$

$$\Rightarrow \int_e^\infty \frac{k}{x(\ln(x))^{k+1}}\,dx = \lim_{t\to\infty} 1 - \frac{1}{(\ln(t))^k} = 1$$

$\uparrow$ $k > 0$

Q50 Capital Value $= \int_0^\infty 10000\,e^{0.04t} \cdot e^{-0.12t}\,dt$

$$= \lim_{b\to\infty} \int_0^b 10000\,e^{-0.08t}\,dt = \lim_{b\to\infty} \left.\frac{10000}{-0.08}e^{-0.08t}\right|_0^b$$

$$= \lim_{b \to \infty} \frac{10000}{0.08} - \frac{10000}{0.08} e^{-0.08b} = \frac{10000}{0.08} \text{ dollars.}$$