Homework 5

§9.1

Q1/ $u = x^2+4 \Rightarrow \frac{du}{dx} = 2x \Rightarrow dx = \frac{du}{2x}$

$$\Rightarrow \int 2x(x^2+4)^5\,dx = \int (x^2+4)^5\,du = \int u^5\,du = \frac{1}{6}u^6 + C$$

$$= \frac{1}{6}(x^2+4)^6 + C.$$

Q4/ $u = x^2+2x+3 \Rightarrow \frac{du}{dx} = 2x+2 \Rightarrow dx = \frac{du}{2x+2}$

$$\Rightarrow \int (x^2+2x+3)^6(x+1)\cdot dx = \frac{1}{2}\int (x^2+2x+3)^6\,du = \frac{1}{2}\int u^6\,du$$

$$= \frac{1}{14}u^7 + C = \frac{1}{14}(x^2+2x+3)^7 + C$$

Q7/ $u = 4-x^2 \Rightarrow \frac{du}{dx} = -2x \Rightarrow dx = \frac{du}{-2x}$

$$\Rightarrow \int x\sqrt{4-x^2}\,dx = -\frac{1}{2}\int \sqrt{u}\,du = -\frac{1}{2}\cdot\frac{2}{3}u^{3/2} + C$$

$$= \frac{-1}{3}(4-x^2)^{3/2} + C$$

Q8 $u = 1 + \ln(x) \Rightarrow \frac{du}{dx} = \frac{1}{x} \Rightarrow dx = x\,du$

$\Rightarrow \int \frac{(1+\ln(x))^3}{x}\,dx = \int u^3\,du = \frac{1}{4}u^4 + C = \frac{1}{4}(1+\ln(x))^4 + C$

Q13 $u = \ln(2x) \Rightarrow \frac{du}{dx} = \frac{2}{2x} = \frac{1}{x} \Rightarrow dx = x\,du$

$\Rightarrow \int \frac{\ln(2x)}{x}\,dx = \int u\,du = \frac{1}{2}u^2 + C = \frac{1}{2}(\ln(2x))^2 + C$

Q14 Let $u = \ln\sqrt{x} = \frac{1}{2}\ln(x) \Rightarrow \frac{du}{dx} = \frac{1}{2x} \Rightarrow dx = 2x\,du$

$\Rightarrow \int \frac{\ln\sqrt{x}}{x}\,dx = 2\int u\,du = u^2 + C = (\ln\sqrt{x})^2 + C$

Q21 $u = x^3 - 3x^2 + 1 \Rightarrow \frac{du}{dx} = 3x^2 - 6x \Rightarrow dx = \frac{du}{3x^2-6x}$

$\int \frac{x^2-2x}{x^3-3x^2+1}\,dx = \frac{1}{3}\int \frac{1}{u}\,du = \frac{1}{3}\ln|u| + C = \frac{1}{3}\ln|x^3-3x^2+1| + C$

Q23 Let $u = -x^2 \Rightarrow \frac{du}{dx} = -2x \Rightarrow dx = \frac{du}{-2x}$

$\Rightarrow \int 8x\,e^{-x^2}\,dx = -4\int e^u\,du = -4e^u + C = -4e^{-x^2} + C$

Q32 $u = e^{x} - e^{-x} \Rightarrow \frac{du}{dx} = e^{x} + e^{-x} \Rightarrow dx = \frac{du}{e^{x} + e^{-x}}$

$$\Rightarrow \int \frac{e^{x} + e^{-x}}{e^{x} - e^{-x}}\,dx = \int \frac{1}{u}\,du = \ln|u| + C = \ln|e^{x} - e^{-x}| + C$$

Q35 $$\int \frac{1}{1+e^{x}}\,dx = \int \frac{e^{-x}}{e^{-x}+1}\,dx$$

$u = e^{-x} + 1 \Rightarrow \frac{du}{dx} = -e^{-x} \Rightarrow dx = \frac{du}{-e^{-x}} \Rightarrow$

$$\int \frac{e^{-x}}{e^{-x}+1}\,dx = -\int \frac{1}{u}\,du = -\ln|u| + C = -\ln|e^{-x}+1| + C$$

Q37 $f'(x) = \frac{x}{\sqrt{x^2+9}}$

Let $u = x^2 + 9 \Rightarrow \frac{du}{dx} = 2x \Rightarrow dx = \frac{du}{2x} \Rightarrow$

$$\int \frac{x}{\sqrt{x^2+9}}\,dx = \frac{1}{2}\int \frac{1}{\sqrt{u}}\,du = \sqrt{u} + C = \sqrt{x^2+9} + C$$

$\sqrt{4^2+9} + C = 8 \Rightarrow C = 3$

$\Rightarrow f(x) = \sqrt{x^2+9} + 3$

Q43

$u = \sin(x) \Rightarrow \frac{du}{dx} = \cos(x) \Rightarrow dx = \frac{du}{\cos(x)}$

$\Rightarrow \int \sin(x)\cos(x)\,dx = \int u\,du = \frac{1}{2}u^2 + C = \frac{1}{2}\sin^2(x) + C$

Q50

$\int \cot(x)\,dx = \int \frac{\cos(x)}{\sin(x)}\,dx$

$u = \sin(x) \Rightarrow \frac{du}{dx} = \cos(x) \Rightarrow dx = \frac{du}{\cos(x)} \Rightarrow$

$\int \frac{\cos(x)}{\sin(x)}\,dx = \int \frac{1}{u}\,du = \ln|u| + C = \ln|\sin(x)| + C.$

§4.2

Q1

$f(x) = x \qquad g(x) = e^{5x}$

$f'(x) = 1 \qquad G(x) = \frac{1}{5}e^{5x} \Rightarrow \int x e^{5x}\,dx = \frac{x}{5}e^{5x} - \int \frac{1}{5}e^{5x}\,dx$

$= \frac{x}{5}e^{5x} - \frac{1}{25}e^{5x} + C$

Q6

$f(x) = x^2 \qquad g(x) = e^x$

$f'(x) = 2x \qquad G(x) = e^x \Rightarrow \int x^2 e^x\,dx = x^2 e^x - 2\int x e^x\,dx$

$f(x) = x \qquad g(x) = e^x$

$f'(x) = 1 \qquad G(x) = e^x \Rightarrow \int x e^x \, dx = x e^x - e^x + \text{constat}$

$\Rightarrow \int x^2 e^x \, dx = x^2 e^x - 2x e^x + 2 e^x + C$

Q7 $f(x) = x \qquad g(x) = \frac{1}{\sqrt{x+1}}$

$f'(x) = 1 \qquad G(x) = 2\sqrt{x+1} \Rightarrow \int \frac{x}{\sqrt{x+1}} \, dx = 2x\sqrt{x+1} - \int 2\sqrt{x+1} \, dx$

$= 2x\sqrt{x+1} - \frac{4}{3}(x+1)^{3/2} + C$

Q15 $\sqrt{x} \ln \sqrt{x} = \frac{1}{2}\sqrt{x} \ln(x)$

$f(x) = \ln(x) \qquad g(x) = \frac{1}{2}\sqrt{x}$

$f'(x) = \frac{1}{x} \qquad G(x) = \frac{1}{3} x^{3/2} \Rightarrow \int \sqrt{x} \ln \sqrt{x} \, dx = \frac{1}{3} x^{3/2} \ln(x) - \frac{1}{3}\int \sqrt{x} \, dx$

$= \frac{1}{3} x^{3/2} \ln(x) - \frac{2}{9} x^{3/2} + C$

Q22 $u = \ln(x) \Rightarrow \frac{du}{dx} = \frac{1}{x} \Rightarrow dx = x \, du \Rightarrow$

$\int \frac{\ln(\ln(x))}{x} \, dx = \int \ln(u) \, du$

$f(u) = \ln(u) \qquad g(u) = 1$

$f'(u) = \frac{1}{u} \qquad G(u) = u \Rightarrow \int \ln(u) \, du = u \ln(u) - \int 1 \, du = u\ln(u) - u + C$

$\Rightarrow \int \frac{\ln(\ln(x))}{x}\,dx = \ln(x)\ln(\ln(x)) - \ln(x) + C$

Q38

$f(x) = x \quad g(x) = e^{-x/3}$

$f'(x) = 1 \quad G(x) = -3e^{-x/3}$

$\Rightarrow \int x e^{-\frac{x}{3}}\,dx = -3xe^{-x/3} + 3\int e^{-x/3}\,dx$

$= -3x e^{-x/3} - 9e^{-x/3} + C$

$f(0) = 6 \Rightarrow -9 + C = 6 \Rightarrow C = 15 \Rightarrow$

$f(x) = -3x e^{-x/3} - 9e^{-x/3} + 15$

Q39

$f(x) = xe^{x} \quad g(x) = \frac{1}{(x+1)^2}$

$f'(x) = xe^{x} + e^{x} = (x+1)e^{x} \quad G(x) = \frac{-1}{(x+1)}$

$\Rightarrow \int \frac{xe^{x}}{(x+1)^2}\,dx = -\frac{xe^{x}}{x+1} + \int e^{x}\,dx = \frac{-xe^{x}}{x+1} + e^{x} + C$

Q40 $u = x^4 \Rightarrow \frac{du}{dx} = 4x^3 \Rightarrow dx = \frac{du}{4x^3} \Rightarrow$

$\int x^7 e^{x^4}\, dx = \frac{1}{4}\int u e^u\, du$

$f(u) = u \quad g(u) = e^u$

$f'(u) = 1 \quad G(u) = e^u \Rightarrow \int u e^u\, du = ue^u - e^u + c$

$\Rightarrow \int x^7 e^{x^7}\, dx = \frac{1}{4} x^4 e^{x^4} - \frac{1}{4} e^{x^4} + c$

§9.3

Q4 $u = x^2+1 \Rightarrow \frac{du}{dx} = 2x \Rightarrow dx = \frac{du}{2x}$

$\Rightarrow \int \frac{2x}{\sqrt{x^2+1}}\, dx = \int \frac{1}{\sqrt{u}}\, du = 2\sqrt{u} + c = 2\sqrt{x^2+1} + c$

$\Rightarrow \int_0^1 \frac{2x}{\sqrt{x^2+1}}\, dx = 2\sqrt{x^2+1}\Big|_0^1 = 2\sqrt{2} - 2.$

Q5 $f(x) = x \quad g(x) = \frac{1}{\sqrt{x+1}}$

$f'(x) = 1 \quad G(x) = 2\sqrt{x+1} \Rightarrow \int \frac{x}{\sqrt{x+1}}\, dx = 2x\sqrt{x+1} - 2\int \sqrt{x+1}\, dx$

$$= 2x\sqrt{x+1} - \frac{4}{3}(x+1)^{3/2} + C$$

$$\Rightarrow \int_0^3 \frac{x}{\sqrt{x+1}}\,dx = \left(2x\sqrt{x+1} - \frac{4}{3}(x+1)^{3/2}\right)\Big|_0^3$$

$$= \left(6\sqrt{4} - \frac{4}{3}4^{3/2}\right) + \frac{4}{3}$$

$$= 12 - \frac{32}{3} + \frac{4}{3} = \frac{8}{3}$$

Q17

$$u = \sin(x) \Rightarrow \frac{du}{dx} = \cos(x) \Rightarrow dx = \frac{du}{\cos(x)}$$

$$\Rightarrow \int e^{\sin(x)} \cdot \cos(x)\,dx = \int e^u\,du = e^u + C = e^{\sin(x)} + C$$

$$\Rightarrow \int_0^{\pi} e^{\sin(x)} \cdot \cos(x)\,dx = e^{\sin(x)}\Big|_0^{\pi} = e^0 - e^0 = 0$$

Q18

$f(x) = x \qquad g(x) = \sin(\pi x)$

$f'(x) = 1 \qquad G(x) = -\frac{1}{\pi}\cos(\pi x)$

$$\Rightarrow \int x\sin(\pi x)\,dx = \frac{-1}{\pi}x\cos(\pi x) + \frac{1}{\pi}\int \cos(\pi x)\,dx$$

$$= \frac{-1}{\pi} x \cos(\pi x) + \frac{1}{\pi^2} \sin(\pi x) + C$$

$$\Rightarrow \int_0^1 x \sin(\pi x)\, dx = \frac{-1}{\pi} x \cos(\pi x) + \frac{1}{\pi^2} \sin(\pi x) \Big|_0^1$$

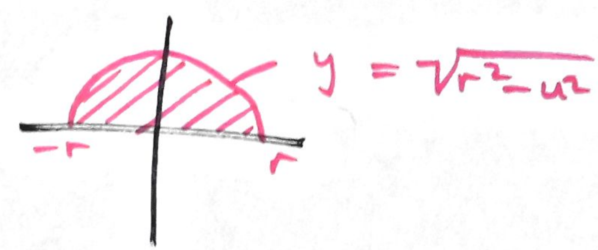
$$= \frac{-1}{\pi} \cos(\pi) = \frac{1}{\pi}.$$

Q21

$$u = \sin(x) \Rightarrow \frac{du}{dx} = \cos(x) \Rightarrow dx = \frac{du}{\cos(x)}$$

$$\Rightarrow \int_{-\pi/2}^{\pi/2} \sqrt{1 - \sin^2(x)}\, \cos(x)\, dx = \int_{-1}^{1} \sqrt{1-u^2}\, du = \frac{\pi}{2}$$

$$\left(\int_{-r}^{r} \sqrt{r^2 - u^2}\, du = \frac{\pi r^2}{2} : \right)$$



Q23

$$u = x + 3 \Rightarrow \frac{du}{dx} = 1 \Rightarrow dx = du$$

$$\Rightarrow \int_{-6}^{0} \sqrt{9 - (x+3)^2}\, dx = \int_{-3}^{3} \sqrt{9 - u^2}\, du = \frac{\pi \cdot 3^2}{2} = \frac{9\pi}{2}.$$

Q25 $u = 4 - x^2 \Rightarrow \frac{du}{dx} = -2x \Rightarrow dx = \frac{du}{-2x}$

$$\Rightarrow \int x\sqrt{4-x^2}\,dx = -\frac{1}{2}\int \sqrt{u}\,du = -\frac{1}{3}u^{3/2} + C$$

$$= \frac{-1}{3}(4-x^2)^{3/2} + C$$

$$\text{Total Area} = \left(\int_0^2 x\sqrt{(4-x^2)}\,dx\right) + \left(-\int_{-2}^0 x\sqrt{4-x^2}\,dx\right)$$

$$= \frac{-1}{3}(4-x^2)^{3/2}\Big|_0^2 - \frac{-1}{3}(4-x^2)^{3/2}\Big|_{-2}^0$$

$$= \frac{8}{3} + \frac{8}{3} = \frac{16}{3}.$$