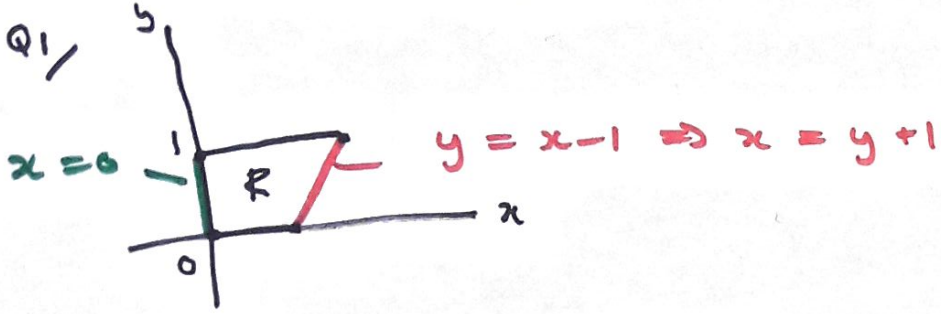
Further Double Integrals :

Q1/

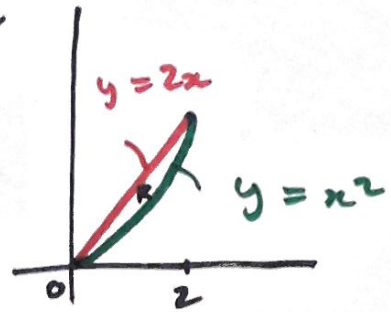



$$\Rightarrow \iint_R xy\,dx\,dy = \int_0^1 \left(\int_0^{y+1} xy\,dx\right)dy = \int_0^1 \left(\frac{x^2}{2}y\Big|_0^{y+1}\right)dy$$

$$= \int_0^1 \frac{y}{2}(y+1)^2\,dy = \frac{1}{2}\int_0^1 y^3 + 2y^2 + y\,dy = \frac{1}{2}\left(\frac{y^4}{4} + \frac{2}{3}y^3 + \frac{y^2}{2}\right)\Big|_0^1$$

$$= \frac{1}{2}\left(\frac{1}{4} + \frac{2}{3} + \frac{1}{2}\right) = \frac{13}{24}$$

Q2/




$$\iint_R x^2y^2\,dx\,dy = \int_0^2 \left(\int_{x^2}^{2x} x^2y^2\,dy\right)dx$$

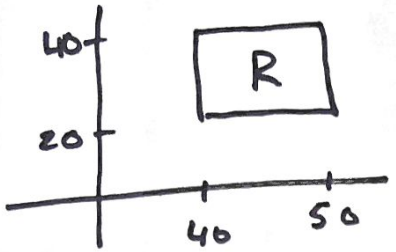
$$= \int_0^2 \left(\frac{x^2y^3}{3}\Big|_{x^2}^{2x}\right)dx = \int_0^2 \frac{8x^5}{3} - \frac{x^8}{3}\,dx$$

$$= \frac{8}{18}x^6 - \frac{x^9}{27}\Big|_0^2 = \frac{8\cdot 2^6}{18} - \frac{2^9}{27} = \frac{512}{54}$$

Q3/

$40 \le x \le 50$

$20 \le y \le 40$




$$\Rightarrow \text{Area}(R) = (40-20)(50-40) = 200$$

$$\iint_R 400x^{0.3}y^{0.7}\,dx\,dy = \int_{20}^{40}\left(\int_{40}^{50} 400x^{0.3}y^{0.7}\,dx\right)dy$$

$$= \int_{20}^{40}\left(400y^{0.7}\int_{40}^{50} x^{0.3}\,dx\right)dy = \int_{20}^{40} 400y^{0.7}\left(\frac{1}{1.3}(50^{1.3}-40^{1.3})\right)dy$$

$$= 400\cdot\frac{1}{1.3}(50^{1.3}-40^{1.3})\int_{20}^{40} y^{0.7}\,dy$$

$$= 400\cdot\left(\frac{1}{1.3}\right)(50^{1.3}-40^{1.3})\left(\frac{1}{1.7}\right)(40^{1.7}-20^{1.7})$$

$$\Rightarrow \text{Average production} = \frac{400}{200}\cdot\frac{1}{1.3}\cdot\frac{1}{1.7}(50^{1.3}-40^{1.3})(40^{1.7}-20^{1.7})$$

$$\simeq 13492.7$$

## 58.1

Q1 $30^\circ = \frac{30}{360} \times 2\pi = \frac{\pi}{6}$ rad.

$120^\circ = \frac{2\pi}{3}$ rad

$315^\circ = \frac{7\pi}{4}$ rad.

Q3 $450^\circ = \frac{450}{360} \times 2\pi = \frac{5\pi}{2}$ rad.

$-210^\circ = -\frac{7\pi}{6}$ rad.

$-90^\circ = \frac{-\pi}{2}$ rad

Q6 $t = -\frac{3\pi}{2}$ rad

Q8 $t = 4\pi + \frac{\pi}{2} = \frac{9\pi}{2}$ rad

Q11 $t = \frac{2}{6} \times 2\pi = \frac{2\pi}{3}$ rad

Q17 Check back of book.

## 58.2

Q1 $\sin(t) = \frac{1}{2}$, $\cos(t) = \frac{\sqrt{3}}{2}$

Q3 $\sin(t) = \frac{2}{\sqrt{13}}$, $\cos(t) = \frac{3}{\sqrt{13}}$




Q6

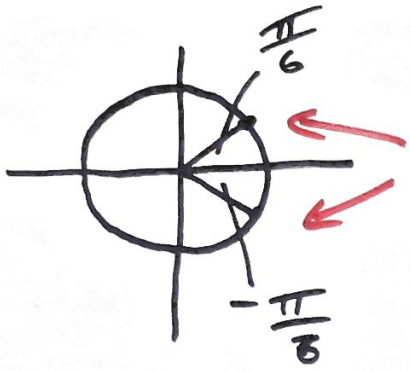

$\Rightarrow \sin(t) = \frac{5}{\sqrt{34}}$, $\cos(t) = \frac{3}{\sqrt{34}}$

Q12


$\Rightarrow \cos(t) = 0.8$

$\sin(t) = -0.6$

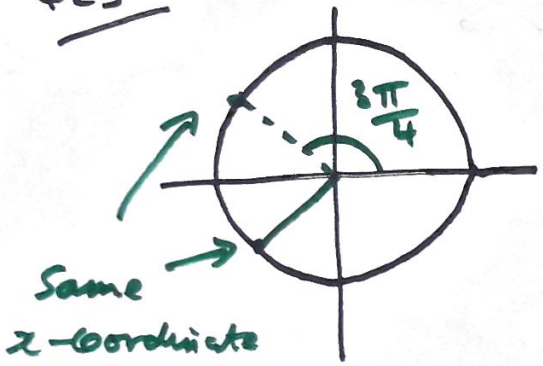
Q21




$\Rightarrow \cos\left(\frac{\pi}{6}\right) = \cos\left(-\frac{\pi}{6}\right)$

$\Rightarrow t = \frac{\pi}{6}$

Q23




$\Rightarrow \cos\left(\frac{3\pi}{4}\right) = \cos\left(\frac{5\pi}{4}\right) \Rightarrow t = \frac{3\pi}{4}$

§8.3

Q1/ $\frac{dy}{dt} = 4\cos(4t)$

Q5/ $\frac{dy}{dt} = -6\sin(3t)$

Q9/ $\frac{dy}{dt} = \cos(t) - t\sin(t)$

Q13/ $\frac{dy}{dx} = \frac{1}{2\sqrt{x-1}} \cdot \cos\left(\sqrt{x-1}\right)$

Q15 $\frac{dy}{dx} = \frac{1}{2\sqrt{\sin(x-1)}} \cdot \cos(x-1)$

Q23 $\frac{dy}{dx} = 2\cos(2x)\cos(3x) - 3\sin(2x)\sin(3x)$

Q27 $\frac{dy}{dx} = \frac{-\sin(x)}{\cos(x)} = -\tan(x)$

Q33 $y(\pi/2) = 3\sin(\pi/2) + \cos(\pi) = 3 - 1 = 2$

$\frac{dy}{dx} = 3\cos(x) - 2\sin(2x) \Rightarrow \left.\frac{dy}{dx}\right|_{x=\pi/2} = 3\cos(\pi/2) - 2\sin(\pi) = 0$

$\Rightarrow$ Tangent is $y = 2$.

Q39 $\int \cos(x) - \sin(x)\,dx = \sin(x) + \cos(x) + C$

Q45 Area $= \int_0^{\pi/2} \cos(t)\,dt = \sin(t)\Big|_0^{\pi/2} = \sin(\pi/2) - \sin(0) = 1$

Q47

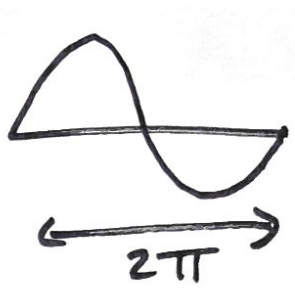
a) $-1 \le \cos(t) \le 1 \Rightarrow -1 \le \cos(6t) \le 1 \Rightarrow 100 - 20 \le 100 + 20\cos(6t) \le 100 + 20$

↑ values are attained ↗

$\Rightarrow$ Max $= 120$

Min $= 80$

b) Wavelength of $\cos(t) = 2\pi$ :

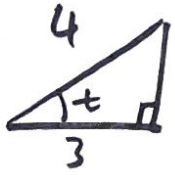



Wavelength of $\cos(6t) = \frac{2\pi}{6}$


$$\Rightarrow \text{BPM} = \frac{60}{(2\pi/6)} = \frac{360}{2\pi} .$$

§8.4

Q4




$$\Rightarrow \tan(t) = \frac{\sqrt{7}}{3} , \sec(t) = \frac{1}{\cos(t)} = \frac{1}{\frac{3}{4}} = \frac{4}{3}$$

Q10 As in Q12 / from 8.2 $\sin(t) = -0.6$ , $\cos(t) = 0.8$

$$\Rightarrow \tan(t) = \frac{-0.6}{0.8} = \frac{-3}{4}$$

Q13 $f'(t) = -(\cos(t))^{-2} \cdot (-\sin(t)) = \frac{\sin(t)}{\cos^2(t)}$

Q27 $\frac{dy}{dx} = \sec^2(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}}$

Q26 $\frac{dy}{dx} = 3e^{3x}\tan(2x) + 2e^{3x}\sec^2(2x)$

Q32 $\frac{dy}{dt} = \frac{\sec^2(t)}{\tan(t)} = \frac{1}{\sin(t)\cos(t)}$

Q37 $\int_{-\pi/4}^{\pi/4} \sec^2(x)\,dx = \tan(x)\Big|_{-\frac{\pi}{4}}^{\pi/4} = \tan(\pi/4) - \tan(-\pi/4)$

$= 2\tan(\pi/4) = 2.$