# HW2 Solutions

## §7.2

Q5/ $\frac{\partial f}{\partial x} = \frac{1}{y} - \frac{y}{x^2}$, $\frac{\partial f}{\partial y} = \frac{-x}{y^2} + \frac{1}{x}$

Q8/ $\frac{\partial f}{\partial x} = \frac{e^x}{1+e^y}$, $\frac{\partial f}{\partial y} = e^x \cdot e^y \cdot -(1+e^y)^{-2} = \frac{-e^{x+y}}{(1+e^y)^2}$

Q10/ $\frac{\partial f}{\partial x} = \frac{y}{xy} = \frac{1}{x}$, $\frac{\partial f}{\partial y} = \frac{x}{xy} = \frac{1}{y}$

Q11/ $$\frac{\partial f}{\partial x} = \frac{1\cdot(x+y) - 1\cdot(x-y)}{(x+y)^2} = \frac{2y}{(x+y)^2}$$

$$\frac{\partial f}{\partial y} = \frac{-1\cdot(x+y) - 1\cdot(x-y)}{(x+y)^2} = \frac{-2x}{(x+y)^2}$$

Q14/ $f(p,q) = 1 - p - pq \Rightarrow \frac{\partial f}{\partial p} = -1 - q$, $\frac{\partial f}{\partial q} = -p$

Q15/ $\frac{\partial f}{\partial x} = \frac{2xy}{z}$, $\frac{\partial f}{\partial y} = \frac{x^2}{z}$, $\frac{\partial f}{\partial z} = \frac{-(1+x^2y)}{z^2}$

Q18/ $\frac{\partial f}{\partial x} = \frac{y}{z}$, $\frac{\partial f}{\partial y} = \frac{x}{z}$, $\frac{\partial f}{\partial z} = \frac{-xy}{z^2}$

Q14/ $\frac{\partial f}{\partial x} = 2x + 2y + 3 \Rightarrow \frac{\partial f}{\partial x}(2,-3) = 4 - 6 + 3 = 1$

$\frac{\partial f}{\partial y} = 2x + 2y + 5 \Rightarrow \frac{\partial f}{\partial y}(2,-3) = 4 - 6 + 5 = 3$

Q24/ $\frac{\partial f}{\partial x} = e^y + 4x^3 y$ , $\frac{\partial f}{\partial y} = x e^y + x^4 + 3y^2$

$\Rightarrow \frac{\partial^2 f}{\partial x^2} = 12x^2 y$ , $\frac{\partial^2 f}{\partial y^2} = x e^y + 6y$ , $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x} = e^y + 4x^3$

Q27 $f(p_1, p_2)$ = number of people riding bus

$p_1$: price of bus, $p_2$: price of train

$f(p_1, p_2)$ = increasing function in $p_2 \Rightarrow \frac{\partial f}{\partial p_2} > 0$
(increase price of train $\Rightarrow$ more on bus)

$f(p_1, p_2)$ = decreasing function in $p_1 \Rightarrow \frac{\partial f}{\partial p_1} < 0$
(increase price of bus $\Rightarrow$ less on bus)

Q29 $f(p_1, p_2)$ = demand for MP3 players

$p_1$: price of player, $p_2$: price of audio file

$g(p_1, p_2)$ = demand for audio file

$f(p_1, p_2)$ = decreasing function in $p_2$ $\Rightarrow$ $\dfrac{\partial f}{\partial p_2} < 0$

(increasing audio file cost $\Rightarrow$ less demand for player)

$g(p_1, p_2)$ = decreasing function in $p_1$ $\Rightarrow$ $\dfrac{\partial g}{\partial p_1} < 0$

(increase price at player $\Rightarrow$ less demand for audio file)

Q34 $\dfrac{\partial f}{\partial x} = 45\, x^{-\frac{1}{4}} y^{1/4}$ , $\dfrac{\partial f}{\partial y} = 15\, x^{3/4} y^{-\frac{3}{4}}$

$\Rightarrow$ $x \dfrac{\partial f}{\partial x} + y \dfrac{\partial f}{\partial y} = 45 \cdot x^{3/4} y^{1/4} + 15\, x^{3/4} y^{1/4} = 60\, x^{3/4} y^{1/4} = f(x,y)$

ie for any $x = a$, $y = b$ $f(a,b) = a \dfrac{\partial f}{\partial x}(a,b) + b \dfrac{\partial f}{\partial y}(a,b)$.

§7.3

Q2/ $\dfrac{\partial f}{\partial x} = x-3$ , $\dfrac{\partial f}{\partial y} = 2y+2$

$\begin{array}{l} x-3=0 \\ 2y+2=0 \end{array} \Rightarrow \begin{array}{l} x=3 \\ y=-1 \end{array} \Rightarrow (3,-1)$ potential max/min

Q6/ $\dfrac{\partial f}{\partial x} = 2x+5$ , $\dfrac{\partial f}{\partial y} = -3y^2+12$

$\begin{array}{l} 2x+5=0 \\ -3y^2+12=0 \end{array} \Rightarrow \begin{array}{l} x=\frac{-5}{2} \\ y=\pm 2 \end{array} \Rightarrow (\frac{-5}{2},2), (\frac{-5}{2},2)$ potential max/min

Q10/ $\dfrac{\partial f}{\partial x} = x + 2y - 1$ , $\dfrac{\partial f}{\partial y} = 2x+6y+2$

$\begin{array}{l} x+2y-1=0 \\ 2x+6y+2=0 \end{array} \Rightarrow x = 1-2y \Rightarrow 2(1-2y)+6y+2=0 \Rightarrow 2y+4=0 \Rightarrow y=-2$

$\Rightarrow x=5 \Rightarrow (5,-2)$ is min location.

Q11/ $\dfrac{\partial f}{\partial x} = 6x-6y$ , $\dfrac{\partial f}{\partial y} = -6x+3y^2-9 \Rightarrow$

$\dfrac{\partial^2 f}{\partial x^2} = 6$ , $\dfrac{\partial^2 f}{\partial y^2} = 6y$ , $\dfrac{\partial^2 f}{\partial x \partial y} = -6 \Rightarrow D(x,y) = 36y-36$

$D(3,3) = 36 \cdot 3 - 36 > 0$ , $\dfrac{\partial^2 f}{\partial x^2}(3,3) = 6 > 0 \Rightarrow$ min at $(3,3)$

$D(-1,-1) = -72 < 0 \Rightarrow$ saddle at $(-1,-1)$.

Q12 $\frac{\partial f}{\partial x} = 6y^2 - 6x^2$ , $\frac{\partial f}{\partial y} = 12xy - 12y^3$

$\Rightarrow \frac{\partial^2 f}{\partial x^2} = -12x$ , $\frac{\partial^2 f}{\partial y^2} = 12x - 36y^2$ , $\frac{\partial^2 f}{\partial x \partial y} = 12y$

$\Rightarrow D(x,y) = (-12x)(12x - 36y^2) - (12y)^2$

$D(0,0) = 0 \Rightarrow$ inconclusive

$D(1,1) = 144 > 0$ , $\frac{\partial^2 f}{\partial x^2}(1,1) = -12 < 0 \Rightarrow (1,1)$ max

$D(1,-1) = 144 > 0$, $\frac{\partial^2 f}{\partial x^2}(1,-1) = -12 < 0 \Rightarrow (1,-1)$ max

Q17 $\frac{\partial f}{\partial x} = 2x - 2y$ , $\frac{\partial f}{\partial y} = -2x + 8y$

$2x - 2y = 0$

$-2x + 8y = 0 \Rightarrow y = x \Rightarrow 6y = 0 \Rightarrow y = 0 \Rightarrow x = 0$

$\Rightarrow (0,0)$ only critical point

$\frac{\partial^2 f}{\partial x^2} = 2$ , $\frac{\partial^2 f}{\partial y^2} = 8$ , $\frac{\partial^2 f}{\partial x \partial y} = -2 \Rightarrow D(x,y) = 2 \cdot 8 - (-2)^2 = 12$

$\Rightarrow D(0,0) = 12 > 0$ , $\frac{\partial^2 f}{\partial x^2}(0,0) = 2 > 0 \Rightarrow (0,0)$ gives relative min

Q24/ $\frac{\partial f}{\partial x} = 3x^2 - 2y$ , $\frac{\partial f}{\partial y} = -2x + 4$

$$\begin{matrix} 3x^2 - 2y = 0 \\ -2x + 4 = 0 \end{matrix} \Rightarrow x = 2 \Rightarrow 12 - 2y = 0 \Rightarrow y = 6 \Rightarrow (2,6) \text{ only critical point}$$

$$\frac{\partial^2 f}{\partial x^2} = 6x \,, \quad \frac{\partial^2 f}{\partial y^2} = 0 \,, \quad \frac{\partial^2 f}{\partial x \partial y} = -2 \Rightarrow D(x,y) = -4$$

$\Rightarrow$ $D(2,6) = -4 < 0$ $\Rightarrow$ $(2,6)$ a saddle point.

Q29/ $f(x,y,z) = xyz$ (objective)

$z + (2x+2y) = 84$ (constraint) $x, y, z > 0$

$$\Rightarrow z = 84 - 2x - 2y$$

$$\Rightarrow xyz = 84xy - 2x^2y - 2xy^2 = h(x,y)$$

$$\begin{matrix} \frac{\partial h}{\partial x} = 84y - 4xy - 2y^2 = 0 \\ \frac{\partial h}{\partial y} = 84x - 2x^2 - 4xy = 0 \end{matrix} \Rightarrow \begin{matrix} 84 - 4x - 2y = 0 \\ 84 - 2x - 4y = 0 \end{matrix}$$

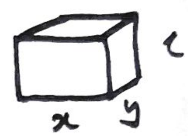
$$\Rightarrow y = 42 - 2x \Rightarrow 84 - 2x - 4(42 - 2x) = 0 \Rightarrow x = 14 \Rightarrow y = 14$$

$$\Rightarrow z = 28$$

Assuming a max volume is attained it must be at $(14, 14, 28)$.

**Q30/**

Objective : $2xy + 2yz + 2xz$




Constraint : $xyz = 1000 \quad ; \; x, y, z > 0$

$$\Rightarrow \quad z = \frac{1000}{xy} \quad \Rightarrow \quad 2xy + 2yz + 2xz = 2xy + \frac{2000}{x} + \frac{2000}{y} = f(x,y)$$

$$\Rightarrow \quad \frac{\partial f}{\partial x} = 2y - \frac{2000}{x^2} \quad , \quad \frac{\partial f}{\partial y} = 2x - \frac{2000}{y^2}$$

$$2y - \frac{2000}{x^2} = 0$$

$$2x - \frac{2000}{y^2} = 0$$

$$\Rightarrow \quad y = \frac{1000}{x^2} \quad \Rightarrow \quad 2x - \frac{2000}{\left(\frac{1000}{x^2}\right)^2} = 0$$

$$\Rightarrow \quad 2x - \frac{2x^4}{1000} = 0 \quad \Rightarrow \quad x^3 = 1000 \quad \Rightarrow \quad x = 10 \quad \Rightarrow \quad y = 10 \quad \Rightarrow \quad z = 10$$

$\Rightarrow$ Min surface area is when $x = 10, \; y = 10, \; z = 10$.

**Q31/** Profit $= P(x,y) = 10x + 9y - 400 - 2x - 3y - \frac{3}{100}x^2 - \frac{xy}{100} - \frac{3}{100}y^2$

$$= 8x + 6y - 400 - \frac{3}{100}x^2 - \frac{xy}{100} - \frac{3}{100}y^2$$

$$\frac{\partial P}{\partial x} = 8 - \frac{6}{100}x - \frac{y}{100} \quad , \quad \frac{\partial P}{\partial y} = 6 - \frac{x}{100} - \frac{6}{100}y$$

$$8 - \frac{6}{100}x - \frac{y}{100} = 0 \qquad 800 - 6x - y = 0$$

$$6 - \frac{x}{100} - \frac{6}{100}y = 0 \Rightarrow 600 - x - 6y = 0 \Rightarrow y = 800 - 6x \Rightarrow 600 - x - 6(800 - 6x) = 0$$

$$\Rightarrow 35x - 4200 = 0 \Rightarrow x = \frac{4200}{35} \Rightarrow y = 800 - \frac{4200}{35} \cdot 6$$

Assuming profits can be maximized this is when it must happen.

Q34 Profit $= P(x,y) = R(x,y) - p_1 x - p_2 y$

$$\frac{\partial P}{\partial x} = \frac{\partial R}{\partial x} - p_1 = 0 \qquad \frac{\partial R}{\partial x} = p_1$$

$$\Rightarrow$$

$$\frac{\partial P}{\partial y} = \frac{\partial R}{\partial y} - p_2 = 0 \qquad \frac{\partial R}{\partial y} = p_2$$

Must occur when profits maximized.