This exam consists of 5 questions. Answer the questions in the spaces provided.

1. Compute the following integrals:

(a) (10 points)

$$\int xe^{2x}dx.$$

**Solution:**

$f(x) = x$ $\quad g(x) = e^{2x}$

$f'(x) = 1$ $\quad G(x) = \frac{1}{2}e^{2x}$ $\Rightarrow \int xe^{2x}dx = \frac{1}{2}xe^{2x} - \int \frac{1}{2}e^{2x}dx$

$= \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C$

(b) (10 points)

$$\int \frac{x^2}{(x^3+2)^3}dx$$

**Solution:**

$u = x^3+2 \Rightarrow \frac{du}{dx} = 3x^2 \Rightarrow dx = \frac{du}{3x^2} \Rightarrow$

$\int \frac{x^2}{(x^3+2)^3}dx = \frac{1}{3}\int \frac{1}{u^3}du = \frac{-1}{6}\cdot\frac{1}{u^2} + C$

$= \frac{-1}{6(x^3+2)^2} + C.$

2. (20 points) A company projects that over the next year they will have a continuous income stream with income rate $5000t^2$ dollars per year. If they intend to invest their income in an account with a 50% interest rate, what is the present value of the company's earning over the next year?

**Solution:**

$f(t) = 5000t^2$

$r = 0.5$

$\Rightarrow$ Present Value over $[0,1]$ $= \int_0^1 5000t^2 e^{-\frac{1}{2}t}\,dt$

$f(t) = t^2$ $\quad g(t) = e^{-\frac{1}{2}t}$

$f'(t) = 2t$ $\quad G(t) = -2e^{-\frac{1}{2}t}$

$\Rightarrow \int t^2 e^{-\frac{1}{2}t}\,dt = -2t^2 e^{-\frac{1}{2}t} + 4\int t e^{-\frac{1}{2}t}\,dt$

$f(t) = t$ $\quad g(t) = e^{-\frac{1}{2}t}$

$f'(t) = 1$ $\quad G(t) = -2e^{-\frac{1}{2}t}$

$\Rightarrow \int t e^{-\frac{1}{2}t}\,dt = -2te^{-\frac{1}{2}t} + 2\int e^{-\frac{1}{2}t}\,dt$

$= -2te^{-\frac{1}{2}t} - 4e^{-\frac{1}{2}t} + C$

$\Rightarrow \int t^2 e^{-\frac{1}{2}t}\,dt = -2t^2 e^{-\frac{1}{2}t} - 8te^{-\frac{1}{2}t} - 16e^{-\frac{1}{2}t} + C$

$\Rightarrow$ Present Value $= 5000(-2t^2 - 8t - 16)e^{-\frac{1}{2}t}\Big|_0^1$

$= \$5000\left(16 - 26e^{-\frac{1}{2}}\right)$

3. (20 points) Find a general solution to the following differential equation:

$$2xy' + y = 4x\ln(x) \quad \Rightarrow \quad x > 0$$

**Solution:**

$2xy' + y = 4x\ln(x) \Rightarrow y' + \frac{1}{2x}y = 2\ln(x)$

$\Rightarrow a(x) = \frac{1}{2x}, \quad b(x) = 2\ln(x)$

Let $A(x) = \frac{1}{2}\ln(x) = \ln(\sqrt{x}) \Rightarrow e^{A(x)} = \sqrt{x}$

$(x>0)$

$\Rightarrow y = \frac{1}{\sqrt{x}}\int \sqrt{x} \cdot 2\ln(x)\, dx$

$f(x) = \ln(x) \quad g(x) = 2\sqrt{x}$

$f'(x) = \frac{1}{x} \quad G(x) = \frac{4}{3}x^{3/2}$

$\Rightarrow \int \sqrt{x} \cdot 2\ln(x)\,dx = \frac{4}{3}x^{3/2}\ln(x) - \frac{4}{3}\int \sqrt{x}\,dx$

$= \frac{4}{3}x^{3/2}\ln(x) - \frac{8}{9}x^{3/2} + C$

$\Rightarrow y = \frac{4}{3}x\ln(x) - \frac{8}{9}x + \frac{C}{\sqrt{x}}$

4. (a) (10 points) Find a general solution to the following differential equation:

$$y' = \frac{xe^{-y^2}}{y}$$

**Solution:**

$\frac{e^{-y^2}}{y}$ is never $0$ $\Rightarrow$ no constant solutions

$\int y e^{y^2}\, dy = \int x\, dx$

$u = y^2 \Rightarrow \frac{du}{dy} = 2y \Rightarrow dy = \frac{du}{2y} \Rightarrow \int y e^{y^2} dy = \frac{1}{2}\int e^u\, du$

$= \frac{1}{2}e^u + C = \frac{1}{2}e^{y^2} + C$

$\Rightarrow \frac{1}{2}e^{y^2} = \frac{1}{2}x^2 + C \Rightarrow y^2 = \ln(x^2 + 2C)$

$\Rightarrow y = \pm\sqrt{\ln(x^2 + 2C)}$ is general solution

(b) (5 points) Using part(a) find a solution which satisfies the initial condition

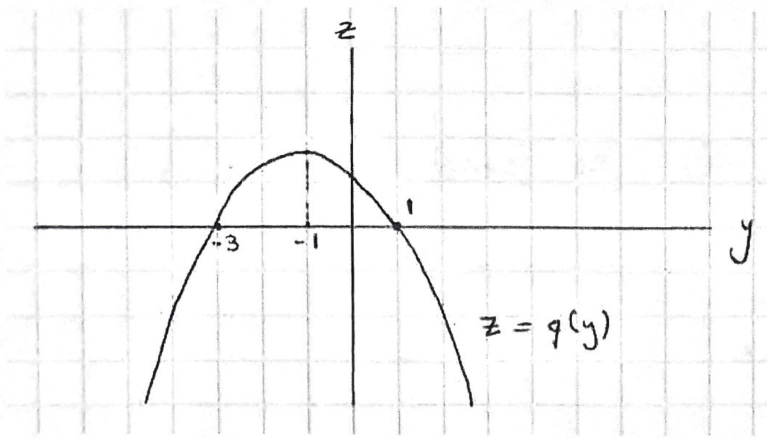
$$y(0) = -1.$$

**Solution:**

$y(0) = -1 \Rightarrow -\sqrt{\ln(2C)} = -1 \Rightarrow \ln(2C) = 1$

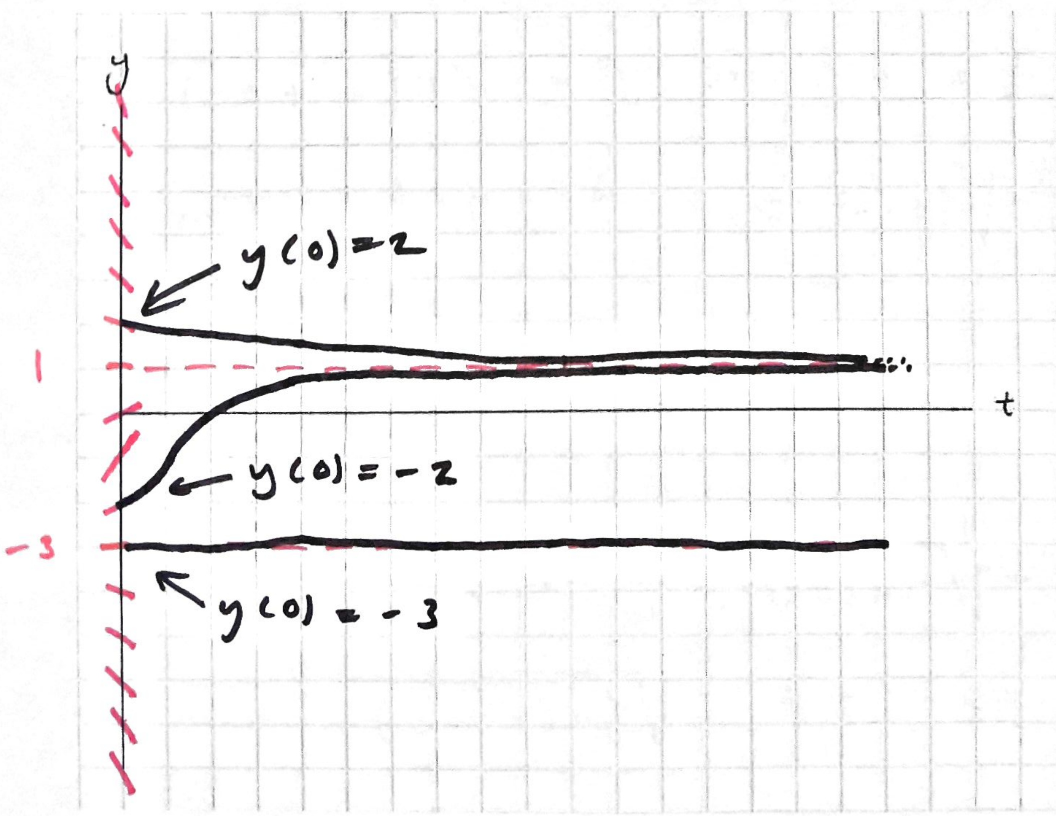
$\Rightarrow 2C = e \Rightarrow y = -\sqrt{\ln(x^2 + e)}$

5. (20 points) Consider the differential equation of the form $y' = q(y)$, where the graph of $z = q(y)$ is as follows:




Sketch a solution for each of the following initial conditions: $y(0) = -3$, $y(0) = -2$ and $y(0) = 2$.

**Solution:**




END OF EXAM