MATH 16B MIDTERM 2 (002) 9AM - 10AM

PROFESSOR PAULIN

DO NOT TURN OVER UNTIL INSTRUCTED TO DO SO.

CALCULATORS ARE NOT PERMITTED

YOU MAY USE YOUR OWN BLANK PAPER FOR ROUGH WORK

SO AS NOT TO DISTURB OTHER STUDENTS, EVERYONE MUST STAY UNTIL THE EXAM IS FINISHED

REMEMBER THIS EXAM IS GRADED BY A HUMAN BEING. WRITE YOUR SOLUTIONS NEATLY AND COHERENTLY, OR THEY RISK NOT RECEIVING FULL CREDIT

Name and section: _____

GSI's name: _____

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This exam consists of 5 questions. Answer the questions in the spaces provided.

1. Compute the following integrals:

(a) (10 points)

$$\int \frac{1}{x\ln(x)}dx.$$

Solution:

$u = \ln(x) \Rightarrow \frac{du}{dx} = \frac{1}{x} \Rightarrow dx = x\,du \Rightarrow$

$\int \frac{1}{x\ln(x)}dx = \int \frac{1}{u}du = \ln|u| + C = \ln|\ln(x)| + C$

(b) (15 points)

$$\int x^2\sin(x)dx.$$

Solution:

$f(x) = x^2 \quad g(x) = \sin(x)$

$f'(x) = 2x \quad G(x) = -\cos(x) \Rightarrow \int x^2\sin(x)dx = -x^2\cos(x) + 2\int x\cos(x)\,dx$

$f(x) = x \quad g(x) = \cos(x)$

$f'(x) = 1 \quad G(x) = \sin(x) \Rightarrow \int x\cos(x)dx = x\sin(x) - \int \sin(x)dx$

$= x\sin(x) + \cos(x) + C$

$\Rightarrow \int x^2\sin(x)dx = -x^2\cos(x) + 2x\sin(x) + 2\cos(x) + C$

2. (a) (20 points) Find a general solution to the following differential equation:

$$\sec(x)y' = \frac{1}{\sqrt{2y+1}}$$

Solution:

never 0 so no constant sols.

$$\sec(x)\,y' = \frac{1}{\sqrt{2y+1}} \Rightarrow \frac{dy}{dx} = \cos(x)\cdot\frac{1}{\sqrt{2y+1}}$$

$$\int \sqrt{2y+1}\,dy = \int \cos(x)\,dx$$

$$\Rightarrow \frac{1}{2}\cdot\frac{2}{3}(2y+1)^{3/2} = \sin(x) + C$$

$$\Rightarrow (2y+1)^{3/2} = 3\sin(x) + 3C$$

$$\Rightarrow y = \frac{(3\sin(x)+3C)^{2/3} - 1}{2}$$ general solution

(b) (5 points) Using part(a) find a solution which satisfies the initial condition

$$y(0) = 4.$$

Solution:

$$y(0) = 4 \Rightarrow 4 = \frac{(3C)^{2/3} - 1}{2} \Rightarrow (3C)^{2/3} = 9$$

$$\Rightarrow 3C = 9^{3/2} = 27 \Rightarrow y = \frac{(3\sin(x)+27)^{2/3} - 1}{2}$$

3. (25 points) Find a general solution to the following differential equation:

$$2xy' + xe^{\sqrt{x}} = y \quad \Rightarrow x > 0$$

Solution:

$2xy' + xe^{\sqrt{x}} = y \Rightarrow y' - \frac{1}{2x}y = -\frac{1}{2}e^{\sqrt{x}}$

$\Rightarrow a(x) = -\frac{1}{2x} \quad b(x) = -\frac{1}{2}e^{\sqrt{x}}$

$A(x) = -\frac{1}{2}\ln|x| = \ln\left(\frac{1}{\sqrt{x}}\right) \Rightarrow e^{A(x)} = \frac{1}{\sqrt{x}}$

$\Rightarrow y(x) = \sqrt{x}\int -\frac{1}{2}\frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

$u = \sqrt{x} \Rightarrow \frac{du}{dx} = \frac{1}{2}\cdot\frac{1}{\sqrt{x}} \Rightarrow dx = 2\sqrt{x}\,du$

$\Rightarrow \int -\frac{1}{2}\frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = -\int e^u du = -e^u + C = -e^{\sqrt{x}} + C$

$\Rightarrow y(x) = -\sqrt{x}e^{\sqrt{x}} + C\sqrt{x}$ is general solution

4. (25 points) A company expects that over t years they will have total continuous income \$3000t. They will invest it in a saving account, resulting in the company having capital value \$60000. What is the annual interest rate of the savings account?

Solution:

$$f(t) = \frac{d}{dt}(3000t) = 3000 \quad \text{- income rate}$$

$$\text{Capital Value} = \int_0^\infty 3000e^{-rt}\,dt$$

$$\int 3000e^{-rt}\,dt = \frac{3000}{-r}e^{-rt} + C$$

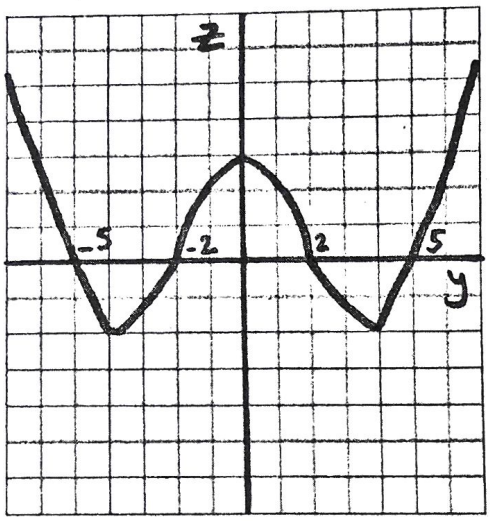
$$\Rightarrow \int_0^b 3000e^{-rt}\,dt = \frac{3000}{-r}e^{-rt}\Big|_0^b = \frac{3000}{r} - \frac{3000}{r}e^{-rb}$$

$$\Rightarrow 60000 = \lim_{b\to\infty} \frac{3000}{r} - \frac{3000}{r}e^{-rb} = \frac{3000}{r}$$

$$\Rightarrow r = \frac{3000}{60000} = 0.05$$

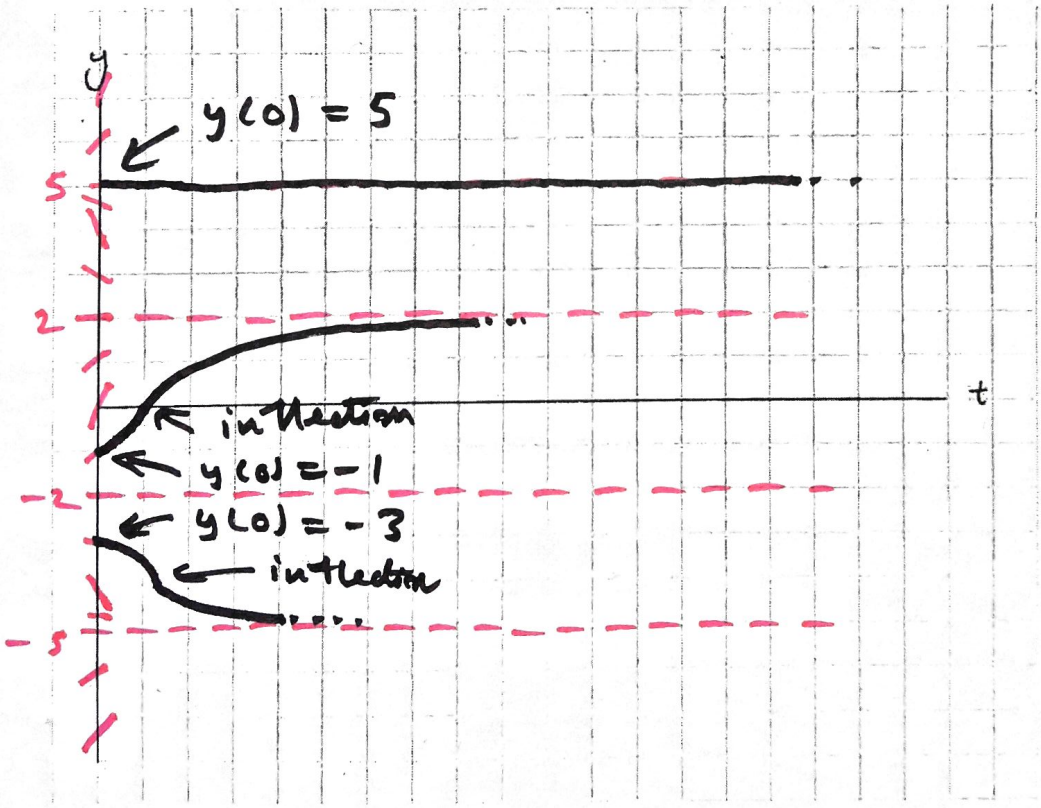
$\Rightarrow$ interest rate is 5%

5. (25 points) Consider the differential equation of the form $y' = q(y)$, where the graph of $z = q(y)$ is as follows:



a solution for each of the following initial conditions: $y(0) = -3$, $y(0) = 5$ and $y(0) = -1$.

Solution:



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