MATH 16B MIDTERM 2 (001) 12.10PM - 1PM

PROFESSOR PAULIN

DO NOT TURN OVER UNTIL INSTRUCTED TO DO SO.

CALCULATORS ARE NOT PERMITTED

YOU MAY USE YOUR OWN BLANK PAPER FOR ROUGH WORK

SO AS NOT TO DISTURB OTHER STUDENTS, EVERYONE MUST STAY UNTIL THE EXAM IS FINISHED

REMEMBER THIS EXAM IS GRADED BY A HUMAN BEING. WRITE YOUR SOLUTIONS NEATLY AND COHERENTLY, OR THEY RISK NOT RECEIVING FULL CREDIT

Name and section: _____

GSI's name: _____

Blank Page

This exam consists of 5 questions. Answer the questions in the spaces provided.

1. Compute the following integrals:

(a) (10 points)

$$\int \tan(x)dx.$$

Solution:

$u = \cos(x) \Rightarrow \frac{du}{dx} = -\sin(x) \Rightarrow dx = \frac{du}{-\sin(x)}$

$\Rightarrow \int \tan(x)dx = \int \frac{\sin(x)}{\cos(x)} dx = -\int \frac{1}{u} du = -\ln|u| + C$

$= -\ln|\cos(x)| + C$

(b) (15 points)

$$\int x^2 e^x dx$$

Solution:

$f(x) = x^2 \quad g(x) = e^x$

$f'(x) = 2x \quad G(x) = e^x \Rightarrow \int x^2 e^x dx = x^2 e^x - 2\int x e^x dx$

$f(x) = x \quad g(x) = e^x$

$f'(x) = 1 \quad G(x) = e^x \Rightarrow \int x e^x dx = xe^x - \int e^x dx$

$= xe^x - e^x + C$

$\Rightarrow \int x^2 e^x dx = x^2 e^x - 2xe^x + 2e^x + C$

2. (a) (20 points) Find a general solution to the following differential equation:

$$\frac{y'}{\sin(x)} = (y^2 - 4y + 4)^2$$

Solution:

$\frac{y'}{\sin(x)} = (y^2-4y+4)^2 = (y-2)^4$ ← $(y-2)^4 = 0 \Rightarrow y = 2$ only constant solution

$\int \frac{1}{(y-2)^4}\,dy = \int \sin(x)\,dx$

$\Rightarrow \frac{1}{-3(y-2)^3} = -\cos(x) + C$

$\Rightarrow (y-2)^3 = \frac{1}{3\cos(x) - 3C} \Rightarrow$

$y = \begin{cases} 2 \\ y = \sqrt[3]{\frac{1}{3\cos(x)-3C}} + 2 \end{cases}$

(b) (5 points) Using part(a) find a solution which satisfies the initial condition

$$y(-3) = 2.$$

Solution:

$y(t) = 2 \Rightarrow y(-3) = 2 \Rightarrow y(t) = 2$ is particular solution

3. (25 points) Find a general solution to the following differential equation:

$$3y + x^3\sqrt{\ln(x)} = xy' \qquad \Rightarrow \quad x > 0$$

Solution:

$3y + x^3\sqrt{\ln(x)} = xy' \Rightarrow y' + \left(\frac{-3}{x}\right)y = x^2\sqrt{\ln(x)}$

$a(x) = \frac{-3}{x} \Rightarrow A(x) = -3\ln(x) = \ln\left(\frac{1}{x^3}\right)$

$b(x) = x^2\sqrt{\ln(x)} \Rightarrow e^{A(x)} = \frac{1}{x^3}$

$\Rightarrow y = x^3 \int \frac{\sqrt{\ln(x)}}{x}\, dx$

$u = \ln(x) \Rightarrow \frac{du}{dx} = \frac{1}{x} \Rightarrow dx = x\,du$

$\Rightarrow \int \frac{\sqrt{\ln(x)}}{x}\, dx = \int \sqrt{u}\, du = \frac{2}{3}u^{3/2} + C$

$= \frac{2}{3}(\ln(x))^{3/2} + C$

$\Rightarrow y = \frac{2x^3}{3}(\ln(x))^{3/2} + Cx^3$

4. (25 points) A company expects to have a steady income of \$10000 per month. They will invest it in a saving account, resulting in the company having capital value \$1200000. What is the annual interest rate of the savings account?

Solution:

$$f(t) = 12 \times 10000 = 120000$$

$$\text{Capital Value} = \int_0^\infty 120000\, e^{-rt}\, dt$$

$$\int 120000\, e^{-rt}\, dt = \frac{120000}{-r} e^{-rt} + C$$

$$\Rightarrow \int_0^b 120000\, e^{-rt}\, dt = \frac{120000}{-r} e^{-rt}\Big|_0^b$$

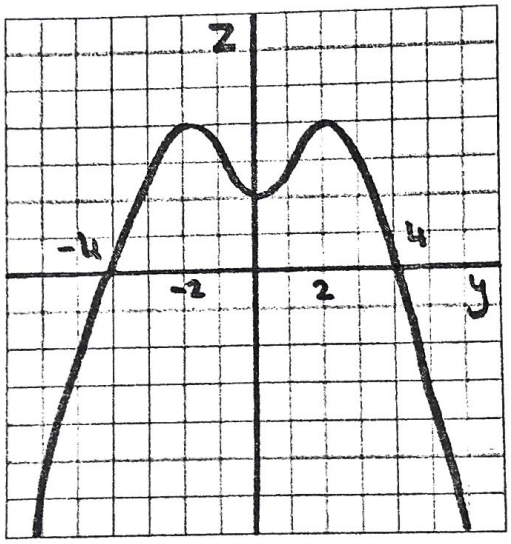
$$= \frac{120000}{r} - \frac{120000}{r} e^{-rb}$$

$$1200000 = \lim_{b \to \infty} \frac{120000}{r} - \frac{120000}{r} e^{-rb} = \frac{120000}{r}$$

$$\Rightarrow r = \frac{120000}{1200000} = 0.1$$

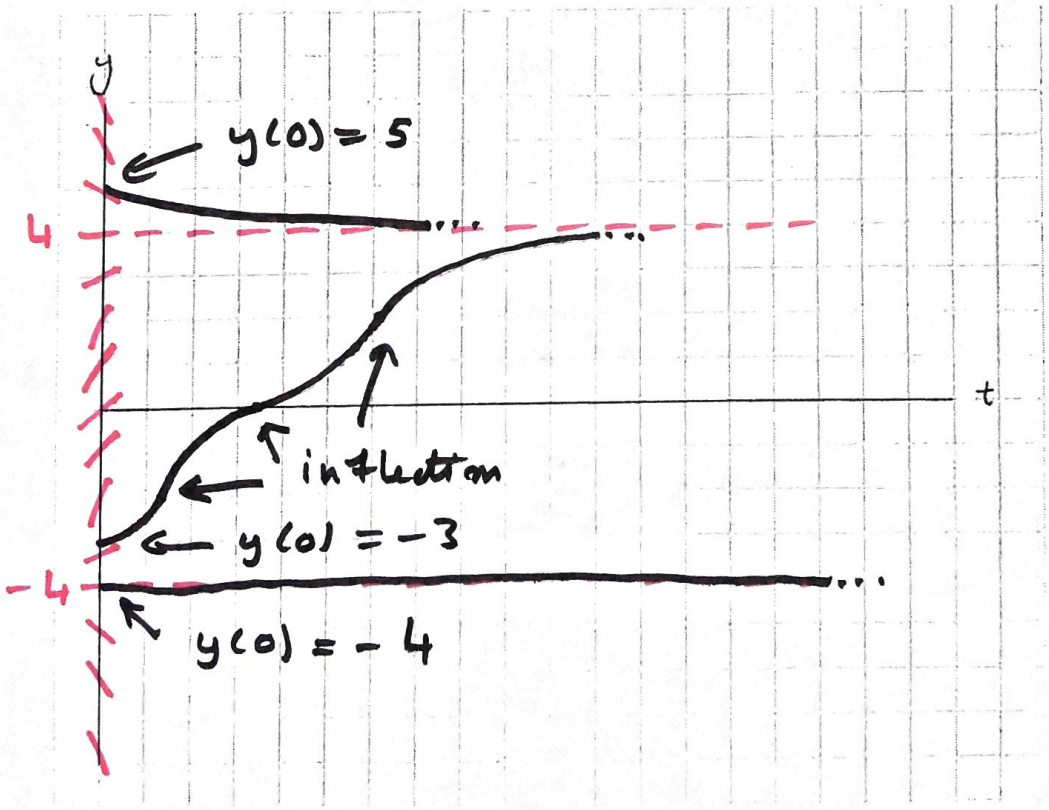
$\Rightarrow$ interest rate is 10%

5. (25 points) Consider the differential equation of the form $y' = q(y)$, where the graph $z = q(y)$ is as follows:



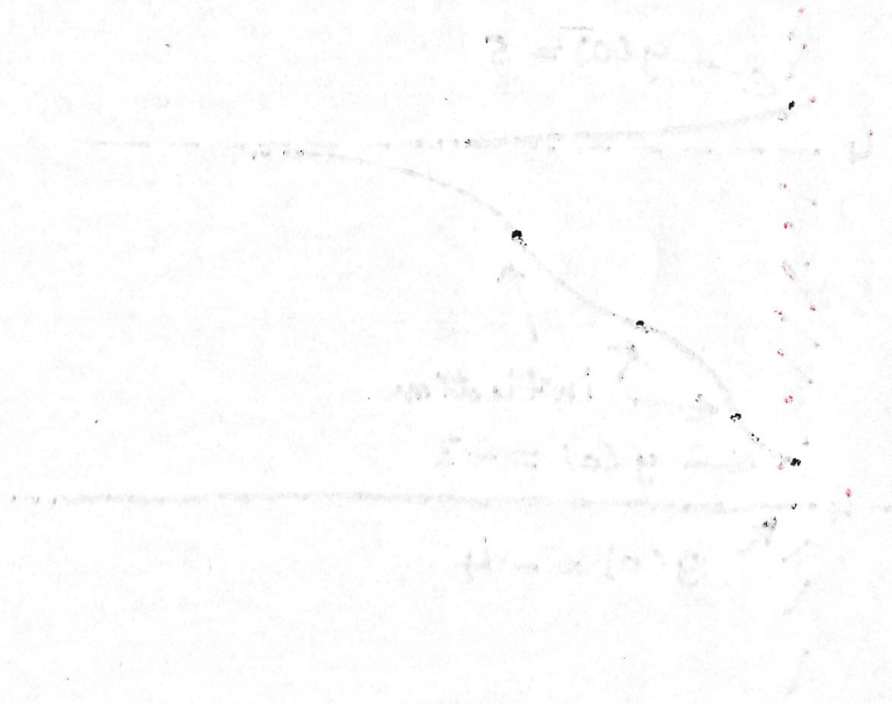
Sketch a solution for each of the following initial conditions: $y(0) = -3$, $y(0) = 5$ and $y(0) = -4$.

Solution:

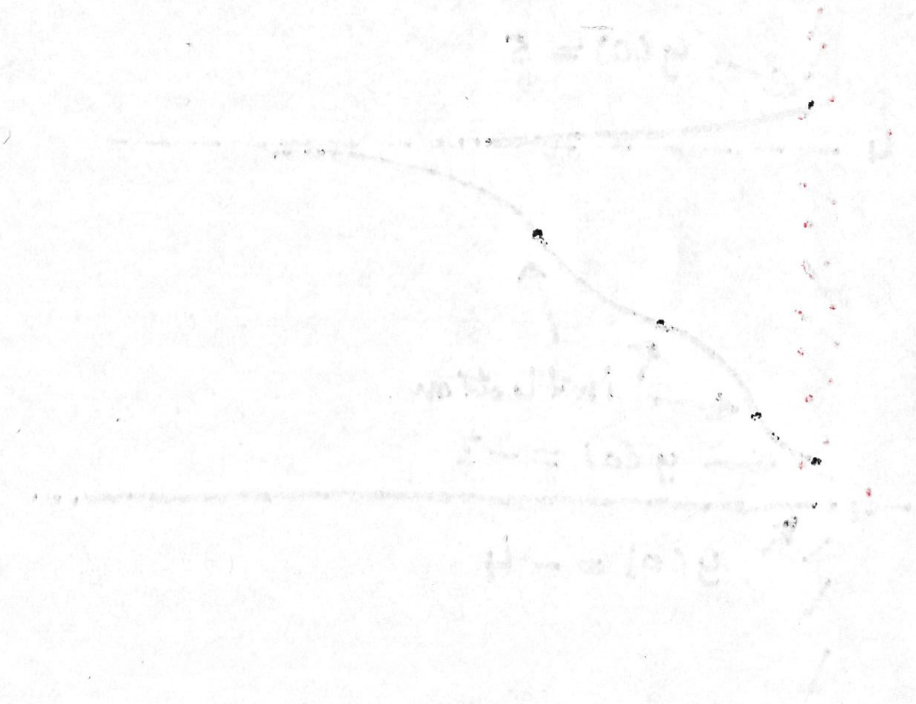


END OF EXAM

Blank Page



Blank Page



Blank Page